MTH 422
Exam 2
Spring 2024

100 points possible. 9 problems at 11 points each, plus 1 free point.

1. X is $U(0, b)$. Find the bias of $\overline{X}$ as an estimator of b. Find an unbiased estimator of b.

2. Prove that $\overline{X}$ is a consistent estimator of the population mean μ.

3. The parameters of the inverse Pareto distribution

$$F(x) = \left(\frac{x}{x+\theta}\right)^{\tau}$$

are to be estimated using the method of moments based on the following data:

10 50 250 550

Calculate the estimate of θ obtained by matching kth moments with $k = -1$ and $k = -2$.

Note: Inverse Pareto $\quad E[X^k] = \dfrac{\theta^k(-k)!}{(\tau-1)\ldots(\tau+k)} \quad$ if k is a negative integer.

4. Let X be $\mathcal{P}(\lambda)$. Find the likelihood function $L(\lambda)$ of a random sample of size 3 of X at the point $(1,\ 2,\ 4)$. Then, using the methods of calculus, find the maximum likelihood estimate of λ at this point.

5. A random sample of claims has been drawn from a Burr distribution with known parameter $\alpha = 1$ and unknown parameters θ and γ. You are given:

(i) 75% of the claim amounts in the sample exceed 200.

(ii) 25% of the claim amounts in the sample exceed 700.

Estimate θ by percentile matching.

Note: Burr $\quad F(x) = 1 - u^{\alpha}, \quad u = \dfrac{1}{1 + (x/\theta)^{\gamma}}$

6. You are given:

- A density function, $f(x) = 7\theta x^6 e^{-\theta x^7}$, $x > 0$
- A random sample of size n from this distribution

Calculate the Rao-Cramer lower bound for the variance of an unbiased estimator of θ.

7. An insurance policy provides coverage for two types of claims. Let X and Y denote the numbers of monthly claims of Type I and Type II, respectively. The joint probability function of X and Y is given by

$$p(x, y) = \frac{10 - 2x - y}{78}, \qquad \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2, 3.$$

Calculate the probability that there are in total at least two claims on this policy in the coming month.

8. In a small metropolitan area, annual losses due to storm, fire, and theft are assumed to be mutually independent, exponentially distributed random variables with respective means 0.9, 1.8, and 3.2.

Calculate the probability that the maximum of these losses exceeds 3.6.

9. A motorist makes three driving errors, each independently resulting in an accident with probability 0.35.

Each accident results in a loss that is exponentially distributed with mean 1.20. Losses are mutually independent and independent of the number of accidents.

The motorist's insurer reimburses 80% of each loss due to an accident.

Calculate the variance of the total unreimbursed loss the motorist experiences due to accidents resulting from these driving errors.