MTH 422
Exam 4
Spring 2022

Take-home exam. Due Wednesday, May 4. Open book, open note, open talk.

1. A group of five individuals with high blood pressure were given a new drug that was designed to lower blood pressure. Systolic blood pressure was measured before and after treatment for each individual, with the following results.

Subject	1	2	3	4	5
Before (y)	168	166	170	160	180
After (x)	145	132	129	135	145

Let μ_D denote the mean of the difference $x - y$. Assume approximate normal distribution, and test $H_0 : \mu_D = 0$ against $H_1 : \mu_D < 0$, using a paired t test with the differences. Use $\alpha = 0.05$. Also use R to find the P-value.

1. Hypotheses

H_0 :

H_1 :

2. Test Statistic

3. Decision Rule ($\alpha =$)

Picture:

Reject H_0 if

Otherwise, fail to reject H_0.

4. Observed Value

5. Conclusion

Reject H_0/Fail to reject H_0 (Circle one)

In English:

2. Let X and Y equal the times in days required for maturation of Guardiola seeds from narrow-leaved and broad-leaved parents, respectively. Assume that X is $N(\mu_X, \sigma_X^2)$ and Y is $N(\mu_Y, \sigma_Y^2)$ and that X and Y are independent. Test the hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$ against the alternative hypothesis $H_1 : \sigma_X^2 > \sigma_Y^2$ if a sample of size $n_X = 21$ yielded $\overline{x} = 18.77$, $s_X^2 = 9.98$ and a sample of size $n_Y = 13$ yielded $\overline{y} = 23.50$, $s_Y^2 = 4.18$. Use $\alpha = 0.05$. Also use R to find the P-value.

1. Hypotheses

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In English:

3. Let Y be $b(85, p)$. To test $H_0 : p = 0.12$ against $H_1 : p < 0.12$, we reject H_0 if and only if $Y \leq 7$.

Answer the following questions using R and the exact binomial distribution, not normal or Poisson approximation.

(a) Determine the significance level α of the test.

(b) Find the probability of the Type II error if, in fact, $p = 0.06$.

4. Let p equal the fraction defective of a certain manufactured item. To test $H_0 : p = 0.1$ against $H_1 : p > 0.1$, we inspect n items selected at random and let Y be the number of defective items in this sample. We reject H_0 if the observed $y \geq c$. Find n and c so that $\alpha = K(0.1) \approx 0.05$ and $K(0.2) \approx 0.90$, where $K(p) = P(Y \geq c\,; p)$. Use normal approximation to binomial, without continuity correction.

5. Suppose X has the density function $f(x) = \dfrac{1}{\theta}$ for $0 < x < \theta$, and zero otherwise. If one observation of X is taken, what are the probabilities α and β of Type I and Type II errors, respectively, in testing the null hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta = 3$, if H_0 is rejected for $X > 1.86$?

6. Let X be the number of successes in n independent trials with the probability of success on each trial equal to p. The null hypothesis $H_0 : p = 0.6$ is to be tested against the alternative hypothesis $H_1 : p > 0.6$. The null hypothesis is rejected if $X = n$. For what value of p is the power function of this test equal to 0.90? (The answer will depend on n.)

7. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state (0 or 1) of a patient, and let Y denote the outcome of the diagnostic test. The joint probability function of X and Y is given by:

$$P[X = 0, Y = 0] = 0.70$$
$$P[X = 0, Y = 1] = 0.18$$
$$P[X = 1, Y = 0] = 0.10$$
$$P[X = 1, Y = 1] = 0.02$$

Calculate $\text{Var}(X \mid Y = 1)$.

8. A motorist makes four driving errors, each independently resulting in an accident with probability 0.3.

Each accident results in a loss that is exponentially distributed with mean 1.4. Losses are mutually independent and independent of the number of accidents.

The motorist's insurer reimburses 55% of each loss due to an accident.

Calculate the variance of the total unreimbursed loss the motorist experiences due to accidents resulting from these four driving errors.