

MTH 422
Exam 3 Take-Home Portion
Spring 2022

Due Monday, April 11.

Some problems ask you to use R. For those problems, write your answers on paper, and also submit a plain text file with a copy of the contents of the R console. I will set Blackboard so that you have the option of submitting multiple files: multiple text files, and a scan or scans of your written solutions. You don't have to submit everything at once. You can upload as you go, if you want.

Open book, open note, open talk.

1. Let X_1, X_2, \dots, X_n be a random sample of a random variable X , for which the density function is $f(x) = 5\theta x^4 e^{-\theta x^5}$, $x > 0$. Calculate the Rao-Cramér lower bound for the variance of an unbiased estimator of θ .

2. Use R. The ozone measurements in Thirsk on 14 randomly selected days were 36, 73, 52, 42, 21, 46, 56, 38, 46, 79, 26, 43, 48, and 64. Assume the ozone count follows a normal distribution. Find a 99% confidence interval for the mean ozone concentration in Thirsk.

3. Use R. A manufacturer is developing a nickel-metal hydride battery that is to be used in cellular telephones in lieu of nickel-cadmium batteries. The director of quality control decides to evaluate the newly developed battery against the widely used nickel-cadmium battery with respect to performance. A random sample of 6 nickel-cadmium batteries and a random sample of 8 of the newly developed nickel-metal hydride batteries are placed in cellular telephones of the same brand and model. The performance measure of interest is the talking time (in minutes) prior to recharging. The results are as follows.

Nickel-Cadmium Battery

54.5 67.8 64.5 70.4 72.5 64.9

Nickel-Metal Hydride Battery

78.3 95.4 69.4 87.3 62.5 85.0 85.3 72.1

Assuming that the performance measures are normally distributed for each battery type, find a 98% confidence interval for the difference in mean performance measures for the two types of batteries. Since the variances might not be equal, use Welch's t .

4. Use R. A sample of 200 transistors selected randomly from the production of Machine A revealed that 12 were defective. A similar sample of 300 from Machine B showed 15 defective. Find a 95% confidence interval for the difference in defective rates for the two machines.

5. You have been given a grant by a women's organization to estimate the proportion of small (10 or fewer employees) incorporated businesses that are owned by women. How many of these small businesses must be surveyed if you want to be 95% confident that the maximum error of the estimate is 0.04?

(a) Assume that nothing is known about the proportion of small incorporated businesses that are owned by women.

(b) Assume that a previous study suggests this proportion is 0.23.

6. Patients with chronic fatigue syndrome were tested, then retested after being treated with fludrocortisone. Listed below are the changes in fatigue after the treatment (based on data from “The Relationship Between Neurally Medicated Hypotension and the Chronic Fatigue Syndrome” by Bou-Holaigah, Rowe, Kan, and Calkins, *Journal of the American Medical Association*, Vol. 274, No. 12). A standard scale from -7 to $+7$ was used, with positive values representing improvements. Assume that the population has an approximately normal distribution. Use $\alpha = 0.05$ to test the claim that the mean change is positive. Use R to find the p -value. Is there evidence that the treatment is effective?

6 5 0 5 6 7 3 3 2 6 5 5 0 6 3 4 3 7

1. Hypotheses

$H_0 :$

$H_1 :$

2. Test Statistic

3. Decision Rule ($\alpha =$)

Picture:

Reject H_0 if

Otherwise, fail to reject H_0 .

4. Observed Value

5. Conclusion

Reject H_0 /Fail to reject H_0 (Circle one)

In English: