MTH 422
Exam 2
Spring 2022

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

1. Take-home problem due Monday. Upload to Blackboard.

2. X is geometric with parameter p. Find the bias of $\overline{X}$ as an estimator of p.

3. Let X be $U(0, b)$. Prove that $2\overline{X}$ is an unbiased estimator of b. Prove that $2\overline{X}$ is a consistent estimator of b.

4. Find the method of moments estimator for the parameter p of a Bernoulli (p) random variable. Then find the method of moments estimate if a sample of size 8 yields $(0, 0, 1, 0, 0, 1, 0, 1)$.

5. Let X be $\mathcal{E}(\lambda)$ where $\lambda = \theta^{-1}$ and θ is the mean. Find the likelihood function $L(\lambda)$ of a random sample of size 5 of X at the point $(10.7,\ 1.9,\ 8.9,\ 23.2,\ 4.8)$. Then, using the methods of calculus, find the maximum likelihood estimate of λ at this point.

6. You are given the following three observations:

$$0.76 \qquad 0.82 \qquad 0.96$$

You fit a distribution with the following density function to the data:

$$f(x) = (p+1)x^p, \quad 0 < x < 1,\ p > -1$$

Use calculus methods to calculate the maximum likelihood estimate of p.

7. A random sample of claims has been drawn from a Burr distribution with known parameter $\alpha = 1$ and unknown parameters θ and γ. You are given:

(i) 75% of the claim amounts in the sample exceed 150.

(ii) 25% of the claim amounts in the sample exceed 650.

Estimate θ by percentile matching.

Note: Burr $\quad F(x) = 1 - u^{\alpha}, \quad u = \dfrac{1}{1 + (x/\theta)^{\gamma}}$