MTH 422
Exam 1
Spring 2022

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

1. Let the distribution of W be $F(7, 4)$. Find the following.

(a) $F_{0.025}(7, 4)$

(b) $F_{0.975}(7, 4)$

(c) $P(0.243 \leq W \leq 14.98)$

2. The profit for a new product is given by $Z = 3X - 4Y - 3$.

X and Y are independent random variables with $\text{Var}(X) = 1$ and $\text{Var}(Y) = 3$.

Calculate $\text{Var}(Z)$.

3. Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.036h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.014h$.

The errors from the two instruments are independent of each other.

Calculate the probability that the average value of the two measurements is within $0.01h$ of the height of the tower.

4. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a four-month period. The following information is known about X and Y:

$E(X) = 60$, $E(Y) = 30$, $\text{Var}(X) = 75$, $\text{Var}(Y) = 50$, $\text{Cov}(X, Y) = 22$.

The totals of hours that different individuals watch movies and sporting events during the four months are mutually independent.

One hundred people are randomly selected and observed for these four months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this four-month period.

Approximate the value of $P[T < 9195]$.

5. If X is a random variable with mean 77 and variance 49, use Chebyshev's inequality to find

(a) a lower bound for $P(62 < X < 92)$;

(b) an upper bound for $P(|X - 77| \geq 21)$.

6. Three letters to different insureds are prepared along with accompanying envelopes. The letters are put into the envelopes randomly.

Calculate the probability that at least one letter ends up in its accompanying envelope.

7. Two fair dice are rolled. Let X be the absolute value of the difference between the two numbers on the dice.

Calculate the probability that $X < 4$.