MTH 422
Quiz 2
Spring 2020

Show all work in a neat and organized fashion. Clearly indicate your answers.
20 points possible.

1. (4 pts.) Let X be exponential with mean $\theta = 1/\lambda$ (so that X has p.d.f. $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$).

Find the likelihood function $L(\lambda)$ of a random sample of size 4 of X at the point $(4.1, 3.8, 6.1, 5.3)$. Using the methods of calculus, showing all work, derive the maximum likelihood estimate of λ at this point.

2. (4 pts.) Let $X_1, X_2, \ldots, X_n$ be a random sample of a random variable X, for which the density function is $f(x) = 6\theta x^5 e^{-\theta x^6}$, $x > 0$. Calculate the Rao-Cramér lower bound for the variance of an unbiased estimator of θ.

3. (4 pts.) Six color picture tubes of a certain type are tested independently for length of life, with results 1213, 1602, 1418, 1539, 1592, and 1343 hours. (Assume that the population has an approximately normal distribution.) Find a 95% confidence interval for the mean lifetime.

4. (4 pts.) A manufacturer is considering installation of a new cost-saving procedure for producing television tuners. There is some question about product quality for the new procedure. A random sample of 610 tuners produced by the new method contains 36 defectives, while 730 tuners produced by the old procedure yields 29 defectives. Find a 95% confidence interval for the difference in defective rates for the two procedures.

5. (4 pts.) Patients with chronic fatigue syndrome were tested, then retested after being treated with fludrocortisone. Listed below are the changes in fatigue after the treatment (based on data from "The Relationship Between Neurally Medicated Hypotension and the Chronic Fatigue Syndrome" by Bou-Holaigah, Rowe, Kan, and Calkins, *Journal of the American Medical Association*, Vol. 274, No. 12). A standard scale from -7 to $+7$ was used, with positive values representing improvements. Use $\alpha = 0.01$ to test the claim that the mean change is positive. (Assume that the population has an approximately normal distribution.) Is there evidence that the treatment is effective?

6 5 0 5 6 7 3 3 2 6 5 5 0 6 3 4 3 7

1. Hypotheses

H_0 :

H_1 :

2. Test Statistic

3. Decision Rule ($\alpha =$)

Picture:

Reject H_0 if

Otherwise, fail to reject H_0.

4. Observed Value

5. Conclusion

Reject H_0/Fail to reject H_0 (Circle one)

In English: