

MTH 422
Exam 2
Spring 2020

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible. Take-home exam.

0. (1 pt.) One free point.
1. (11 pts.) A random sample of claims has been drawn from a Burr distribution with known parameter $\alpha = 1$ and unknown parameters θ and γ . You are given:
- (i) 75% of the claim amounts in the sample exceed 200.
 - (ii) 25% of the claim amounts in the sample exceed 800.

Estimate θ by percentile matching. Use the fact that the cdf for the Burr distribution is $F(x) = 1 - u^\alpha$ where $u = \frac{1}{1 + (x/\theta)^\gamma}$.

2. (11 pts.) The ozone measurements in Thirsk on 15 randomly selected days were 35, 72, 51, 43, 20, 45, 55, 39, 47, 78, 25, 44, 47, 51, and 64. Assume the ozone count follows a normal distribution. Find a 90% confidence interval for the mean ozone concentration in Thirsk.

3. (11 pts.) A machine is used to fill plastic bottles with bleach. Using machine X , the volume of bleach in a bottle is assumed to be $N(\mu_X, 0.010^2)$, measured in liters. Using an independent machine Y , the volume is assumed to be $N(\mu_Y, 0.012^2)$, also measured in liters. A random sample of $n_X = 48$ bottles filled with machine X yielded a mean of $\bar{x} = 2.07$ L, and a random sample of $n_Y = 40$ bottles filled with machine Y yielded a mean of $\bar{y} = 2.01$ L. Find a 99% confidence interval for the difference between the mean fill volumes from the two machines. (Warning: Don't drink bleach.)

4. (11 pts.) You have been given a grant by a women's organization to estimate the proportion of small (10 or fewer employees) incorporated businesses that are owned by women. How many of these small businesses must be surveyed if you want to be 95% confident that the maximum error of the estimate is 0.03?

(a) Assume that nothing is known about the proportion of small incorporated businesses that are owned by women.

(b) Assume that a previous study suggests this proportion is 0.27.

5. (11 pts.) A group of five individuals with high blood pressure were given a new drug that was designed to lower blood pressure. Systolic blood pressure was measured before and after treatment for each individual, with the following results.

Subject	1	2	3	4	5
Before (y)	172	166	170	160	185
After (x)	145	132	129	135	145

Let μ_D denote the mean of the difference $x - y$. Assume approximate normal distribution, and test $H_0 : \mu_D = 0$ against $H_1 : \mu_D < 0$, using a paired t test with the differences. Use $\alpha = 0.05$.

1. Hypotheses

$$H_0 :$$

$$H_1 :$$

2. Test Statistic

3. Decision Rule ($\alpha =$)

Picture:

Reject H_0 if

Otherwise, fail to reject H_0 .

4. Observed Value

5. Conclusion

Reject H_0 /Fail to reject H_0 (Circle one)

In English:

6. (11 pts.) Let X and Y equal the times in days required for maturation of *Guardiola* seeds from narrow-leaved and broad-leaved parents, respectively. Assume that X is $N(\mu_X, \sigma_X^2)$ and Y is $N(\mu_Y, \sigma_Y^2)$ and that X and Y are independent. Test the hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$ against the alternative hypothesis $H_1 : \sigma_X^2 > \sigma_Y^2$ if a sample of size $n_X = 16$ yielded $\bar{x} = 18.57$, $s_X^2 = 9.98$ and a sample of size $n_Y = 13$ yielded $\bar{y} = 24.20$, $s_Y^2 = 3.72$. Use $\alpha = 0.05$.

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$H_0 :$

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In English:

7. (11 pts.) Let Y be $b(70, p)$. To test $H_0 : p = 0.18$ against $H_1 : p < 0.18$, we reject H_0 if and only if $Y \leq 10$.

Answer the following questions using technology and the exact binomial distribution, not normal or Poisson approximation.

(a) Determine the significance level α of the test.

(b) Find the probability of the Type II error if, in fact, $p = 0.12$.

8. (11 pts.) Let p equal the fraction defective of a certain manufactured item. To test $H_0 : p = 0.1$ against $H_1 : p > 0.1$, we inspect n items selected at random and let Y be the number of defective items in this sample. We reject H_0 if the observed $y \geq c$. Find n and c so that $\alpha = K(0.1) \approx 0.05$ and $K(0.2) \approx 0.90$, where $K(p) = P(Y \geq c; p)$. Use normal approximation to binomial, without continuity correction.

9. (11 pts.) Two independent populations X and Y have density functions $f(x) = \lambda x^{\lambda-1}$ for $0 < x < 1$, and $g(y) = \mu y^{\mu-1}$ for $0 < y < 1$, respectively. Let X_1, \dots, X_n , and Y_1, \dots, Y_m , be random samples from X and Y . What is the form of the critical region for the best (Neyman-Pearson) test of $H_0 : \lambda = \mu = 4$ against the alternative hypothesis $H_1 : \lambda = 7, \mu = 2$?