**MTH 422**
**Exam 1**
**Spring 2020**

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

No CAS (e.g., no TI-89, no TI-Nspire).

**1.** (10 pts.) Consider a random variable whose distribution is $F(40, 3)$. Find the following.

(a) $F_{0.05}(40, 3)$

(b) $F_{0.95}(40, 3)$

**2.** (15 pts.) Let $X_1$, $X_2$, and $X_3$ be three independent random variables with binomial distributions $b(8, 1/2)$, $b(6, 1/3)$, and $b(9, 1/6)$, respectively. Find each of the following.

(a) $P(X_1 = 2, X_2 = 2, X_3 = 3)$

(b) $E(X_1 X_2 X_3)$

(c) The mean and the variance of $Y = X_1 + X_2 + X_3$

**3.** (15 pts.) In a given region, the number of tornadoes in a one-week period is modeled by a Poisson distribution with mean 1.75. The numbers of tornadoes in different weeks are mutually independent.

Calculate the probability that fewer than three tornadoes occur in a four-week period.

**4.** (15 pts.) Let $X$ equal the maximal oxygen intake of a human on a treadmill, where the measurements are in milliliters of oxygen per minute per kilogram of weight. Assume that, for a particular population, the mean of $X$ is $\mu = 54.284$ and the standard deviation is $\sigma = 5.8$. Let $\overline{X}$ be the sample mean of a random sample of size $n = 41$. Find $P(52.752 \leq \overline{X} \leq 54.986)$, approximately.

**5.** (15 pts.) You are given the following information:

- $Y = X_1 + \cdots + X_n$ where $n = 100$
- $X_1, \ldots, X_n$ is a random sample from a Gamma distribution with $\alpha = 8$ and $\theta$ unknown

Calculate the value, $c$, to produce an unbiased estimate of $\theta$ from $c * Y$.

**6.** (15 pts.) Let $X$ be $N(\mu, \sigma^2)$ with $\sigma^2 = 16$, and let $(X_1, X_2, X_3, X_4)$ be a random sample of size 4 of $X$. Determine which of these two is the more efficient estimator of $\mu$, showing work to justify your answer.

$$T_1 = \frac{1}{8}X_1 + \frac{3}{8}X_2 + \frac{3}{8}X_3 + \frac{1}{8}X_4 \qquad \text{or} \qquad T_2 = \frac{3}{4}X_1 + \frac{1}{4}X_4$$

**7.** (15 pts.) (a) Let $X$ be $\mathcal{P}(\lambda)$. Find the likelihood function $L(\lambda)$ of a random sample of size 4 of $X$ at the point $(3, 2, 3, 1)$.

(b) Let $X$ be $G(p)$. Find the likelihood function $L(p)$ of a random sample of size 3 of $X$ at the point $(5, 4, 9)$.