MTH 498
Exam 1
Spring 2014

1. Let $(X_1, X_2, \ldots, X_5)$ be a random sample from an $N(20, 64)$ normal distribution. Let

$$W = \sum_{i=1}^{5} (X_i - 20)^2.$$

Find $P(W < 53.184)$.

2. Let X be $U(a - \frac{1}{5}, a + 4)$. Find the method of moments estimator for a.

3. Let X be $N(\mu, 16)$, and $(X_1, X_2, X_3, X_4, X_5)$ be a random sample of size 5 of X.

(a) Find the bias of T_1 as an estimator of μ, if

$$T_1 = 0.1X_1 + 0.6X_3 + 0.1X_4.$$

(b) Determine whether T_2 or T_3 is the more efficient estimator of μ, where

$$T_2 = 0.6X_2 + 0.1X_3 + 0.2X_4 + 0.1X_5 \qquad \text{and} \qquad T_3 = 0.3X_1 + 0.2X_2 + 0.5X_4.$$

4. Let X equal the weight in grams of a miniature candy bar. Assume that the distribution of X is $N(24.43, 2.20)$. Let $\overline{X}$ be the sample mean of a random sample of 30 observations of X.

(a) Find $E(\overline{X})$ and $\mathrm{Var}(\overline{X})$.

(b) Find $P(24.17 \le \overline{X} \le 24.82)$.

5. If the moment-generating function of X is

$$M(t) = e^{10t + 2t^2},$$

then find the probability $P[15.364 \le (X - 10)^2 \le 26.54]$.

6. Use moment-generating functions to prove that if X is $\chi^2(n_1)$, Y is $\chi^2(n_2)$, and X and Y are independent, then $X + Y$ is $\chi^2(n_1 + n_2)$.

7. Let X_1 and X_2 be independent random variables with p.d.f.'s $f_1(x_1) = 2x_1$, $0 < x_1 < 1$, and $f_2(x_2) = 4x^2$, $0 < x_2 < 1$, respectively. Compute

(a) $P(0.5 < X_1 < 1 \text{ and } 0.4 < X_2 < 0.8)$.

(b) $E[X_1^2 X_2^3]$.

8. Let X be exponential with mean $\theta = 1/\lambda$ (so that X has p.d.f. $f(x) = \lambda e^{-\lambda x}$ for $x \ge 0$).

(a) Find the likelihood function $L(\lambda)$ of a random sample of size 4 of X at the point $(4, 3.8, 6.1, 5)$. Using the methods of calculus, derive the maximum likelihood estimate of λ at this point.

(b) Find the likelihood function $L(\lambda)$ of a random sample of size n of X at the point $(x_1, \ldots, x_n)$. Using the methods of calculus, derive the maximum likelihood estimate of λ at this point.