MTH 498
Exam 2
Spring 2008

Equally weighted problems. Your best problem counts double.

1. Let X and Y have a bivariate normal distribution with $\mu_X = 12$, $\mu_Y = 64$, $\sigma_X^2 = 25$, $\sigma_Y^2 = 36$, and the correlation coefficient $\rho_{XY} = 0.7$.

(a) Compute $P(X \geq 16)$.

(b) Compute $P(X \geq 16 \mid Y = 50)$.

2. Let X be $N(\mu, 9)$, and $(X_1, X_2, X_3, X_4, X_5)$ be a random sample of size 5 of X.

(a) Find the bias of T_1 as an estimator of μ, if

$$T_1 = \frac{1}{10}X_1 + \frac{2}{5}X_3 + \frac{1}{5}X_4.$$

(b) Determine whether T_2 or T_3 is the more efficient estimator of μ, where

$$T_2 = \frac{1}{5}X_1 + \frac{3}{5}X_2 + \frac{1}{5}X_4 \qquad \text{or} \qquad T_3 = \frac{1}{16}X_2 + \frac{1}{16}X_3 + \frac{3}{4}X_4 + \frac{1}{8}X_5.$$

3. Let X be $U(b - \frac{3}{4}, b + 2)$. Find the method of moments estimator for b.

4. A population has a density function given by

$$f(x) = \frac{1}{6\theta^4}x^3 e^{-x/\theta}$$

for $x > 0$, where $\theta > 0$ is unknown. Use calculus methods to find the maximum likelihood estimator of θ, based on a random sample of size n.

5. Researchers studying commercial air-filtering systems for noise pollution reported the following sample results for mean noise level. Construct a 95% confidence interval for the difference in the mean noise levels, assuming that the populations are normally distributed with equal variances.

Unit A:	$n = 6$	$\bar{x} = 91.3$	$s = 1.1$
Unit B:	$n = 8$	$\bar{x} = 87.5$	$s = 3.7$

6. From past experience it is known that the mean compressive strength (measured in units of 100 pounds per square inch) of bricks is normally distributed with unknown mean μ and known variance $\sigma^2 = 36$. The bricks will not be acceptable unless we can be reasonably sure that $\mu > 50$. To test the null hypothesis $H_0 : \mu = 50$ against the alternative hypothesis $H_1 : \mu > 50$, nine bricks will be randomly selected and their compressive strengths recorded. The null hypothesis will be rejected if $\overline{x} > 53.3$, where $\overline{x}$ is the observed mean of the sample.

(a) Find the power function $K(\mu)$ for this test.

(b) What is the significance level of this test?

7. An ice cream supplier claims that among the four most popular flavors, customers have these preference rates: 65% prefer vanilla, 15% prefer chocolate, 12% prefer neapolitan, and 8% prefer vanilla fudge. A random sample of 200 customers produces the results below. At the 0.05 significance level, test the claim that the percentages given by the supplier are correct.

Flavor	Vanilla	Chocolate	Neapolitan	Vanilla Fudge
Number prefer	120	40	18	22

1. Hypotheses

   H_0 :

   H_1 :

2. Test Statistic

3. Decision Rule ($\alpha =$)

   Picture:

   Reject H_0 if

   Otherwise, fail to reject H_0.

4. Observed Value

5. Conclusion

   Reject H_0/Fail to reject H_0 (Circle one)

   In English: