

MTH 498
Exam 1
Spring 2008

Equally weighted problems. Your best problem counts double.

1. Take-home problem is due tomorrow.
2. Jar A contains 3 red and 7 white marbles; Jar B contains 4 red and 2 white marbles. A jar is chosen at random. A marble is selected from the chosen jar and placed in the other jar. A marble is then selected from this second jar. What is the probability it is white?

3. Let X_1 and X_2 have independent distributions $b(4, 0.3)$ and $b(7, 0.3)$. Find the moment generating function of $Y = 2X_1 + 3X_2$.

4. A population random variable X has the distribution

x	1	2
$f_X(x)$	0.9	0.1

(a) Let (X_1, X_2, X_3) be a random sample of size 3 of X . Complete the second column of the table, showing the probability distribution of (X_1, X_2, X_3) .

(x_1, x_2, x_3)	$f(x_1, x_2, x_3)$	\bar{x}	m_2	v	s^2
(1, 1, 1)					
(1, 1, 2)					
(1, 2, 1)					
(1, 2, 2)					
(2, 1, 1)					
(2, 1, 2)					
(2, 2, 1)					
(2, 2, 2)					
	1.000				

(b) Complete the remaining columns above, showing \bar{x} , m_2 , v , and s^2 for each sample point. Recall $m_2 = \frac{1}{n} \sum x_i^2$, $v = m_2 - \bar{x}^2$, and $s^2 = nv/(n - 1)$.

(c) Complete the following table showing the distribution of \bar{X} on the real line.

\bar{x}					
$f_{\bar{X}}(\bar{x})$					1.000

(d) Create a table showing the distribution of the statistics S^2 and V .

5. Let (X_1, X_2, \dots, X_6) be a random sample from the standard normal distribution $N(10, 25)$. Let

$$W = \sum_{i=1}^6 (X_i - 10)^2.$$

Find $P(W > 361.25)$.

6. A random sample is taken from $N(\mu, \sigma^2)$, and the following observations are recorded.

13.1 5.1 18.0 8.7 16.5 9.8 6.8

Find a 90% confidence interval for σ^2 .

7. Let X be the number of alpha particles counted by a Geiger counter during one minute. Assume that the distribution of X is Poisson with a mean of 9658. Determine (approximately) $P(9564 < X < 9712)$.