Math 498
Exam 2

You should use the inside front cover of the textbook as a cheat sheet. You also should use the tables in the back of the textbook.

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

1. (4 pts.) Rearrange these letters to form a word.

D E N C I F C O N E

2. (12 pts.) Let X be Poisson with mean λ.

(a) Find the likelihood function $L(\lambda)$ of a random sample of size 3 of X at the point $(2, 6, 5)$. Using the methods of calculus, derive the maximum likelihood estimate of λ at this point.

(b) Find the likelihood function $L(\lambda)$ of a random sample of size n of X at the point $(x_1, \ldots, x_n)$. Using the methods of calculus, derive the maximum likelihood estimate of λ at this point.

(c) Find the method of moments estimator for the parameter λ.

3. (12 pts.) Let X be geometric with parameter p (and mean $1/p$).

(a) Find the likelihood function $L(\lambda)$ of a random sample of size 4 of X at the point $(6, 6, 5, 2)$. Using the methods of calculus, derive the maximum likelihood estimate of λ at this point.

(b) Find the likelihood function $L(\lambda)$ of a random sample of size n of X at the point $(x_1, \ldots, x_n)$. Using the methods of calculus, derive the maximum likelihood estimate of λ at this point.

(c) Find the method of moments estimator for the parameter λ.

4. (12 pts.) Derive the method of moments estimators for a and b when X is $U(a, b)$.

5. (12 pts.) The ozone measurements in Capitol City on 15 randomly selected days were 35, 72, 51, 43, 20, 45, 55, 39, 47, 78, 25, 43, 46, 50, and 62. Assume the ozone count follows a normal distribution.

(a) Find a 90% confidence interval for the standard deviation of the ozone concentration in Springfield.

(b) Find a 90% confidence interval for σ that has minimal length.

6. (12 pts.) A sample of 200 transistors selected randomly from the production of Machine #1 revealed that 11 were defective. A similar sample of 300 from Machine #2 showed 14 defective.

(a) Find a 95% confidence interval for the difference in defective rates for the two machines.

(b) Find a one-sided 95% confidence interval that gives a lower bound for $p_1 - p_2$.

7. (12 pts.) To test the effectiveness of a lesson, a teacher gives randomly selected students a pretest and a follow-up test. The results are given below.

Student	A	B	C	D	E	F	G	H
Before	6	8	5	4	3	5	4	7
After	9	10	8	7	6	8	7	10

Find a 95% confidence interval for the mean of the differnce in the scores.

8. (12 pts.) Two identical jars, jar A and jar B, contain 10 marbles each. Jar A contains 6 white and 4 red marbles, while jar B contains 3 white and 7 red marbles. A statistician is asked to select one of the jars and then to determine the contents of the jar by drawing five marbles with replacement from the selected jar. The statistician decides to call the jar A unless four or five selected marbles are red, in which case the jar will be called jar B.

(a) In terms of p, the proportion of red marbles in the jar, what would be the null and alternate hypotheses for the test?

(b) What is α, the size of the Type I error?

(c) What is β, the size of the Type II error?

9. (12 pts.) A customer has ordered electrical condensers stipulating a defective rate no higher than 0.035. Let p be the proportion of condensers that are defective. Test the null hypothesis $H_0 : \ p = 0.035$ against the alternative hypothesis $H_1 : p > 0.035$, using a significance level of $\alpha = 0.05$. A random sample of 70 condensers selected from a shipment yields 5 defective condensers. Should the shipment be rejected as not meeting specifications as to the defective rate?

1. Hypotheses

H_0 :

H_1 :

2. Test Statistic

3. Decision Rule ($\alpha =$)

Picture:

Reject H_0 if

Otherwise, fail to reject H_0.

4. Observed Value

5. Conclusion

Reject H_0/Fail to reject H_0 (Circle one)

In English: