Math 498
Exam 1

You should use the inside front cover of the textbook as a cheat sheet. You also should use the tables in the back of the textbook.

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

1. (4 pts.) Rearrange these letters to form a word.

I S T I C S S T A T

2. (12 pts.) Let X_1, X_2, and X_3 be three independent random variables with binomial distributions $b(8,1/2)$, $b(6,1/3)$, and $b(30,1/6)$, respectively. Find the mean and variance of $Y = 5X_1 - 4X_2 + 3X_3$.

3. (12 pts.) Let X_1, X_2, and X_3 be mutually independent random variables with Poisson distributions having means 3, 5, and 2, respectively.

(a) Find the moment-generating function of the sum $Y = X_1 + X_2 + X_3$.

(b) How is Y distributed?

4. (12 pts.) A population random variable X has the distribution

x	0	2
$f_X(x)$	0.2	0.8

(a) Let (X_1, X_2) be a random sample of size 2 of X. Complete the second column of the table, showing the probability distribution of (X_1, X_2).

(x_1, x_2)	$f(x_1, x_2)$	$\overline{x}$
$(0,0)$		
$(0,2)$		
$(2,0)$		
$(2,2)$		
	1.000	

(b) Complete the third column above, showing $\overline{x}$ for each sample point. Notice how this value depends upon the sample point obtained when the sample is taken.

(c) Complete the following table showing the distribution of $\overline{X}$ on the real line.

$\overline{x}$	
$f_{\overline{X}}(\overline{x})$	

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5. (12 pts.) Let $\overline{X}$ be the sample mean of a random sample of size 36 from an exponential distribution with mean 4. Approximate $P(3.5 \le \overline{X} \le 5)$.

6. (12 pts.) A nursery man plants 115 cuttings of ivy in a flat. Assume that the probability that an individual cutting will develop roots is 0.9. What is the probability (approximate) that at least 100 cuttings will develop roots? Use a normal approximation, with a continuity correction.

7. (12 pts.) The random variable X is $U(1,5)$. Find the upper bound given by Chebyshev's Inequality for the probability that X differs from its mean by at least 1.5. That is, find an upper bound for $P(|X-\mu| \ge 1.5)$.

8. (12 pts.) X is geometric with parameter p. Find the bias of $\overline{X}$ as an estimator of p.

9. (12 pts.) Recall that if the parent random variable is $N(\mu,\sigma^2)$, then the random variable $\dfrac{(n-1)S^2}{\sigma^2}$ is $\chi^2(n-1)$ and thus has variance $2(n-1)$. Under the condition that X is $N(\mu,\sigma^2)$, show that S^2 is a consistent estimator of the population variance σ^2.

Use either the theorem or the corollary below.

Theorem. If

$$\lim_{n\to\infty} E[T_n] = \tau, \qquad \text{and}$$

$$\lim_{n\to\infty} \sigma^2_{T_n} = 0,$$

then T is a consistent estimator of τ.

Corollary. If T is an unbiased estimator of τ and

$$\lim_{n\to\infty} \sigma^2_{T_n} = 0,$$

then T is a consistent estimator of τ.