MTH 421
Exam 3
Fall 2025

100 points possible. 6 problems at 16 points each, plus 4 free points.

There are 7 problems here. Do at least 6 of them.

If you try all 7, then I'll pick your best 6. No extra credit, though.

1. Rectangular plastic covers for a compact disc tray have specifications regarding length and width. Let X be the length and Y be the width, each measured to the nearest millimeter, of a randomly sampled cover. The joint probability mass function of X and Y is given by the following table.

		Width Y	
		17	18
Length X	129	0.14	0.06
	130	0.48	0.22
	131	0.08	0.02

(a) The area of a cover is given by $A = XY$. Find the probability that the area is less than 2350 mm^2.

(b) Find Var(Y).

2. The random pair (X, Y) has the joint probability mass function given in the following table.

		Y		
		-1	0	1
X	0	$1/20$	$4/20$	0
	1	$5/20$	$2/20$	$1/20$
	2	$3/20$	$2/20$	$2/20$

Find the following.

(a) $p_{Y|X}(y \mid 1)$

(b) $E(Y \mid X = 1)$

(c) $\mathrm{Var}(Y \mid X = 1)$

3. A life insurance agent may be happy or sad. The probability of happines is 0.7. On a day when the agent is happy, the number of policies sold is exponentially distributed with mean 0.5. When the agent is sad, the number of policies sold is exponentially distributed with mean 0.2.

Calculate the variance of the number of policies sold per day.

4. Let $S = \{(x, y) : 0 < y < 1, 0 < x < 8 - 8y\}$. Define

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{4} & \text{if } (x, y) \in S \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal pdf $f_Y(y)$.

(b) Find $f_{X|Y}(x \mid \frac{3}{4})$

5. An actuary analyzes a company's annual personal auto claims, M, and annual commercial auto claims, N. The analysis reveals that $\text{Var}(M) = 3600$, $\text{Var}(N) = 400$, and the correlation between M and N is 0.81.

Calculate $\text{Var}(M + N)$.

6. Let N denote the number of accidents occurring during one month on the northbound side of a highway and let S denote the number occurring on the southbound side.

Suppose that N and S are jointly distributed as indicated in the table.

$N \setminus S$	0	1	2	3 or more
0	0.04	0.06	0.10	0.04
1	0.07	0.24	0.05	0.03
2	0.16	0.03	0.04	0.02
3 or more	0.05	0.04	0.02	0.01

Calculate $\mathrm{Var}(N \mid N + S = 2)$.

7. On Main Street, a driver's speed just before an accident is uniformly distributed on $[4, 26]$. Given the speed, the resulting loss from the accident is exponentially distributed with mean equal to five times the speed.

Calculate the variance of a loss due to an accident on Main Street.