MTH 421
Exam 2
Fall 2025

100 points possible. 8 problems at 12 points each, plus 4 free points.

1. In a casino game, a gambler selects five different numbers from the first twenty positive integers. The casino then randomly draws fourteen numbers without replacement from the first twenty positive integers. The gambler wins the jackpot if the casino draws all five of the gambler's selected numbers.

Calculate the probability that the gambler wins the jackpot.

2. A free-throw shooter repeatedly attempts free throws.

(a) Assume that on each attempt, 0.75 is their probability of making the free throw. Assume the attempts are independent. What is the probability of having the first miss on the 7th attempt or later?

(b) What is the probability that the fourth miss occurs on the 16th attempt?

3. Flaws in a certain type of drapery material appear on the average of one in 350 square feet. If we assume a Poisson distribution, find the probability of at most two flaws appearing in 980 square feet.

4. A company has purchased a policy that will compensate for the loss of revenue due to severe weather events. The policy pays 1000 for each severe weather event in a year after the first three such events in that year. The number of severe weather events per year has a Poisson distribution with mean 2.

Calculate the expected amount paid to this company in one year.

5. The life X (in years) of a voltage regulator of a car has the pdf

$$f(x) = \frac{2x}{25} e^{-x^2/25}, \quad 0 < x < \infty.$$

(a) What is the probability that this regulator will last at least four years?

(b) Given that it has lasted at least four years, what is the conditional probability that it will last at least another one year?

6. The time to failure of a component in an electronic device has an exponential distribution with a median of six hours.

Calculate the probability that the component will work without failing for at least four hours.

7. If Z is $N(0,1)$, find the following.

(a) $P(-1.83 < Z \leq 1.16)$

(b) $P(Z < -0.49)$

(c) $P(|Z| < 1.52)$

8. An insurance policy reimburses a loss up to a benefit limit of 12. The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} 18y^{-3}, & y > 3 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of the benefit paid under the insurance policy.