MTH 421
Exam 1
Fall 2025

100 points possible. 8 problems at 12 points each, plus 4 free points.

1. Prove the following formula.

$$(*) \quad P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

You may use the following formula without prooof.

$$(**) \qquad P(D \cup E) = P(D) + P(E) - P(D \cap E)$$

To prove $(*)$, write $A \cup B \cup C = A \cup (B \cup C)$ and apply $(**)$.

2. A box contains 5 blue marbles and 7 gray marbles. Suppose 3 marbles are selected at random and without replacement. Calculate the probability that the number of blue marbles selected exceeds the number of gray marbles selected.

3. Let A and B be independent events with $P(A) = 0.6$ and $P(B) = 0.4$. Compute the following.

(a) $P(A \cap B)$

(b) $P(A \cup B)$

(c) $P(A' \cup B')$

4. An actuary studying the insurance preferences of automobile owners makes the following conclusions:

(i) An automobile owner is three times as likely to purchase collision coverage as disability coverage.
(ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
(iii) The probability that an automobile owner purchases both collision and disability coverages is 0.25.

What is the probability that an automobile owner purchases neither collision nor disability coverage?

5. A health study tracked a group of persons for five years. At the beginning of the study, 10% were classified as heavy smokers, 30% as light smokers, and 60% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period.

Calculate the probability that the participant was a heavy smoker.

6. A four-sided die has four equally likely outcomes when tossed: 1, 2, 3, and 4.

A pair of four-sided dice are tossed. Let X be the larger of the two outcomes. If the outcomes are tied, use the common value.

Find the pmf of X.

7. Determine the constant c so that $p(x)$ satisfies the conditions of being a pmf for a random variable X.

Then find $E(X)$.

$$p(x) = \begin{cases} \dfrac{x}{c}, & x = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

8. An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment generating function

$$M_X(t) = \frac{1}{(1 - 300t)^5}.$$

Calculate the standard deviation of the claim size for this class of accidents.