MTH 421
Half Exam 4
Fall 2021

Show all work in a neat and organized fashion. Clearly indicate your answers.
50 points possible.

0. (2 pts.) Two free points

1. (12 pts.) Let the joint pmf $f(x, y)$ of X and Y be given by the following.

(x, y)	$f(x, y)$
$(0, 0)$	0.4
$(0, 1)$	0.1
$(1, 0)$	0.2
$(1, 1)$	0.3

Find the conditional probability mass functions $g(x \mid 0)$ and $g(x \mid 1)$. Also find the corresponding means and variances $E(X \mid Y = 0)$, $\text{Var}(X \mid Y = 0)$, $E(X \mid Y = 1)$, and $\text{Var}(X \mid Y = 1)$.

2. (12 pts.) An automobile insurance company divides its policyholders into two groups: good drivers and bad drivers. For the good drivers, the amount of an average claim is 1600, with a variance of 90,000. For the bad drivers, the amount of an average claim is 2500, with a variance of 360,000. Seventy percent of the policyholders are classified as good drivers.

Calculate the variance of the amount of a claim for a policyholder.

3. (12 pts.) Determine the value of c for which the function is a joint pdf of two continuous random variables X and Y.

$$f(x, y) = cye^{2x}, \qquad 0 \leq x \leq y^2, \quad 0 \leq y \leq 1$$

4. (12 pts.) Once a fire is reported to a fire insurance company, the company makes an initial estimate, X, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount, Y, to the claimant. The company has determined that X and Y have the joint density function

$$f(x,y) = \frac{2}{x^2(x-1)} y^{-(2x-1)/(x-1)} \quad x > 1,\ y > 1.$$

Given that the initial claim estimated by the company is 3, determine the probability that the final settlement amount is between 4 and 9.