MTH 421
Half Exam 3
Fall 2021

Show all work in a neat and organized fashion. Clearly indicate your answers.
50 points possible.

1. (10 pts.) The pdf of X is $f(x) = c/x^4$ for $2 < x < \infty$.

(a) Calculate the value of c so that f is a pdf.

(b) Find $E(X)$.

2. (10 pts.) The time to failure of a component in an electronic device has an exponential distribution with a median of eight hours.

Calculate the probability that the component will work without failing for at least ten hours.

3. (10 pts.) If X is normally distributed with a mean of 15 and a variance of 81, find the following.

(a) $P(3 \leq X < 19)$

(b) $P(X > 21)$

(c) $P(|X - 15| < 20)$

4. (10 pts.) An insurance policy reimburses a loss up to a benefit limit of 9. The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} 8y^{-3}, & y > 2 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of the benefit paid under the insurance policy.

5. (10 pts.) A red spinner has three equally likely outcomes: 1, 2, and 3.

A black spinner has four equally likely outcomes: 1, 2, 3, and 4.

Spin both spinners. Let X equal the outcome of the red spinner and let Y equal the sum of the two spinners.

(a) Use a table (or other similar design) to display the joint pmf on the space.

(b) Give the marginal pmf of X in the margin. Likewise for Y.

(c) Are X and Y dependent or independent? Explain briefly.