MTH 421
Quiz 2
Fall 2017

Show all work in a neat and organized fashion. Clearly indicate your answers.
20 points possible.

No CAS (e.g., no TI-89, no TI-Nspire).

1. (5 pts.) The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.0032(25 - x) & \text{for } 0 < x < 25 \\ 0 & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 5, what is the probability that it exceeds 10?

2. (5 pts.) The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 40% of high-risk drivers will be involved in an accident during the first 80 days of a calendar year.

What portion of high-risk drivers are expected to be involved in an accident during the first 110 days of a calendar year?

3. (5 pts.) If X is normally distributed with a mean of 9 and a variance of 49, find the following.

(a) $P(-2 \leq X < 12)$

(b) $P(X > 21)$

(c) $P(|X - 9| < 10)$

4. (5 pts.) Let the joint pmf of X and Y be defined by

$$f(x,y) = \frac{x+3y}{64}, \qquad x = 0,1 \quad y = 1,2,3,4.$$

(a) Find $f_X(x)$, the marginal pmf of X. (If you want, you may simply show this in the margin of a table.)

(b) Find $P(Y > 2X)$.

(c) Find the variance of Y.

Useful formulas for the exponential distribution.

$$S(t) = e^{-\lambda t} \ (t > 0), \qquad S(t) = e^{-t/\theta} \ (t > 0)$$

$$F(t) = 1 - e^{-\lambda t} \ (t > 0), \qquad F(t) = 1 - e^{-t/\theta} \ (t > 0)$$

$$f(t) = \lambda e^{-\lambda t} \ (t > 0), \qquad f(t) = \frac{1}{\theta} e^{-t/\theta} \ (t > 0)$$

$$M(t) = \frac{\lambda}{\lambda - t} \ (t < \lambda), \qquad M(t) = \frac{1}{1 - \theta t} \ (t < \lambda)$$

$$\mu = \frac{1}{\lambda}, \qquad \mu = \theta$$

$$\sigma^2 = \frac{1}{\lambda^2}, \qquad \mu = \theta^2$$

Note $m =$ the median of $X \Rightarrow F(m) = \frac{1}{2} \Rightarrow e^{-\lambda m} = \frac{1}{2} \Rightarrow m = (\ln 2)\mu$.