

MTH 421

Exam 2

Fall 2017

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

No CAS (e.g., no TI-89, no TI-Nspire).

0. (4 pts.) Four free points.

1. (12 pts.) If Z is $N(0, 1)$, find the following.

(a) $P(-2.16 < Z \leq 1.55)$

(b) $P(Z < -0.61)$

2. (12 pts.) The time to failure of a component in an electronic device has an exponential distribution with a median of five hours.

Calculate the probability that the component will work without failing for at least nine hours.

3. (12 pts.) An insurers annual weather-related loss, X , is a random variable with density function

$$f(x) = \begin{cases} \frac{200}{x^3} & \text{for } x > 10 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the 40th percentile of X .

4. (12 pts.) Let the joint pmf of X and Y be defined by

$$f(x, y) = \frac{4x + y}{36}, \quad x = 0, 1 \quad y = 1, 2, 3, 4.$$

(a) Find $f_Y(y)$, the marginal pmf of Y . (If you want, you may simply show this in the margin of a table.)

(b) Find $P(X \leq Y - 3)$.

5. (12 pts.) Let the joint pmf of X and Y be

$$f(x, y) = 1/3, \quad (x, y) \in S = \{(0, 0), (2, 2), (2, 3)\}.$$

(a) Are X and Y independent?

(b) Calculate $\text{Cov}(X, Y)$.

6. (12 pts.) Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} \frac{1}{4}x & \text{for } 0 \leq x \leq 2, 0 \leq y \leq x^2 \\ 0 & \text{otherwise.} \end{cases}$$

Find the marginal density function of Y .

7. (12 pts.) Let $f(x, y) = \frac{3}{4}x$, $0 < x < 2$, $x < y < 2$, be the joint pdf of X and Y . Calculate $E(Y | X = x)$, the conditional mean of Y , given that $X = x$.

8. (12 pts.) Let X and Y have a bivariate normal distribution with $\mu_X = 12$, $\mu_Y = 64$, $\sigma_X^2 = 25$, $\sigma_Y^2 = 36$, and correlation coefficient $\rho = 0.7$.

(a) Compute $P(Y \leq 73)$.

(b) Compute $P(Y \leq 73 \mid X = 8)$.