MTH 421
Exam 1
Fall 2017

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

No CAS (e.g., no TI-89, no TI-Nspire).

1. (10 pts.) A survey of a group's viewing habits over the last year revealed the following information:

(i) 38% watched gymnastics
(ii) 18% watched baseball
(iii) 29% watched soccer
(iv) 8% watched gymnastics and baseball
(v) 10% watched baseball and soccer
(vi) 14% watched gymnastics and soccer
(vii) 6% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

2. (10 pts.) A public health researcher examines the medical records of a group of 859 women who died in 2007 and discovers that 325 of the women died from causes related to heart disease. Moreover, 216 of the 859 women had at least one parent who suffered from heart disease, and, of these 216 women, 103 died from causes related to heart disease.

Determine the probability that a woman randomly selected from this group died of causes related to heart disease, given that neither of her parents suffered from heart disease.

3. (10 pts.) Let A and B be independent events with $P(A) = 0.35$ and $P(B) = 0.7$. Compute the following.

(a) $P(A \cap B)$

(b) $P(A \cap B')$

(c) $P(A' \cap B')$

(d) $P[(A \cup B)']$

(e) $P(A' \cap B)$

4. (10 pts.) There is a new diagnostic test for a disease that occurs in about 2.5% of the population. The test is not perfect, but will detect a person with the disease 98% of the time. It will, however, say that a person without the disease has the disease about 4% of the time. A person is selected at random from the population. What is the conditional probability that the person has the disease, given that the test indicates that this person has the disease?

5. (10 pts.) Find and sketch the line graph for the pmf of the random variable which has the following cdf.

$$F(x) = \begin{cases} 0, & x < -\frac{3}{2} \\ 0.2, & -\frac{3}{2} \leq x < \frac{3}{2} \\ 0.8, & \frac{3}{2} \leq x < \frac{9}{2} \\ 1, & \frac{9}{2} \leq x. \end{cases}$$

6. (10 pts.) In a gambling game, for a \$1 bet it is possible to win \$1, \$2, or \$4 with respective probabilities 75/216, 15/216, and 1/216. One dollar is lost with probability 125/216. Let X equal the payoff for this game and find $E(X)$. Note that when a bet is won, the \$1 that was bet, in addition to the \$1, \$2, or \$4 that is won, is returned to the bettor.

7. (10 pts.) For each question on a multiple-choice test, there are four possible answers, of which exactly one is correct. If a student selects answers at random, give the probability that the first question answered correctly is question 6.

8. (10 pts.) On a seven-question multiple-choice test there are four possible answers for each question, of which one is correct (C) and three are incorrect (I). If a student guesses randomly and independently, find the probability of

(a) Being correct only on questions 2 and 5 (i.e., scoring I, C, I, I, C, I, I).

(b) Being correct on two questions.

9. (10 pts.) A free-throw shooter attempts several free throws until he misses.

(a) If $p = 0.85$ is his probability of making a free throw, what is the probability of having the first miss on the 11th attempt or later?

(b) If he continues shooting until he misses four, what is the probability that the fourth miss occurs on the 26th attempt?

10. (10 pts.) A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 1000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 2.5.

Calculate the expected amount paid to the company under this policy during a one-year period.