Math 421
Exam 2

You should use the inside front cover of the textbook as a cheat sheet.

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

1. (10 pts.) Use the moment-generating function of a gamma distribution to show that $E(X) = \alpha\theta$ and $\text{Var}(X) = \alpha\theta^2$.

2. (10 pts.) A mortgage company has experienced a default rate of 2%. If this rate continues and 45 mortgages are written one month, what is the expected number of defaults? What is the standard deviation of the number of defaults?

3. (10 pts.) Let X be $U(a, b)$. Use the p.d.f. to find $E[X^k]$. (Do not use the moment-generating function.)

4. (10 pts.) A radioactive source emits an average of 40 particles per minute. What is the expected value and standard deviation of the time between emissions? What is the probability that a particle will be emitted before 2 seconds have passed?

5. (10 pts.) The random variable X has the p.d.f.

$$f(x) = \begin{cases} 3x^2, & -1 < x < 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the distribution function (c.d.f.) of X. Sketch the distribution function and the p.d.f. on the same set of axes.

6. (10 pts.) Carpet manufactured by a certain machine has an average of 2 flaws per 10 square yards. What is the probability of no flaws in 12 square yards produced by this machine?

7. (10 pts.) Apples are packaged automatically in 3-pound bags. Suppose that 4% of the time, the bag of apples weighs less than 3 pounds. If you select bags randomly and weigh them in order to find one underweight bag of apples, find the probability that the number of bags that must be selected is at least 20.

8. (10 pts.) Let $f(x) = \frac{1}{2}\sin x$, $0 \le x \le \pi$, be the p.d.f. of X. Find μ and σ^2.

9. (10 pts.) Assume a seed germinates with probability 0.3. What is the probability that in a set of 5 seeds, exactly 3 of them will germinate? What is the probability that at least 3 of the set will germinate?

10. (10 pts.) The p.d.f. of X is $f(x) = 2/x^3$, $1 < x < \infty$. Show that $\text{Var}(X)$ is not finite.