MTH 361
Exam 3
Spring 2025

100 points possible. 8 problems at 12 points each, plus 4 free points.

1. Let $\sigma = (1\ 5\ 6)(1\ 4\ 5\ 3)$ in S_6. Find the index of $\langle \sigma \rangle$ in S_6.

2. Give the order of the element in the factor group.

$$(2,2) + \langle(1,2)\rangle \quad \text{in} \quad (\mathbb{Z}_3 \times \mathbb{Z}_9)/\langle(1,2)\rangle$$

3. Classify the given group according to the fundamental theorem of finitely generated abelian groups.

$$(\mathbb{Z}_2 \times \mathbb{Z}_8)/\langle(0, 6)\rangle$$

4. Determine whether the given map ϕ is a homomorphism. Support your answer. (Thorough proof is not required.)

Let $\phi : \mathbb{Z}_7 \to \mathbb{Z}_2$ be given by

$$\phi(x) = \text{the remainder when } x \text{ is divided by 2, as in the division algorithm.}$$

5. Compute the indicated quantities for the given homomorphism ϕ.

$\operatorname{Ker} \phi$ and $\phi(10)$ for $\phi : \mathbb{Z}_{20} \to S_8$ such that $\phi(1) = (1\ 6)(3\ 5\ 7\ 8)$

6. Give an example of a nontrivial homomorphism ϕ for the given groups, if an example exists. If no such homomorphism exists, explain why that is so.

$$\phi : \mathbb{Z}_{14} \to \mathbb{Z}_{13}$$

7. Prove that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G.

8. Mark each of the following true or false. (No reasons required.)

Recall: A *torsion group* is a group all of whose elements have finite order. A group is *torsion free* if the identity is the only element of finite order.

(a) Every finite group contains an element of every order that divides the order of the group.

(b) It is not possible to have a nontrivial homomorphism of some finite group into some infinite group.

(c) Only subgroups of finite groups can have left cosets.

(d) An inner automorphism of an abelian group must be just the identity map.

(e) Every factor group of a torsion group is a torsion group.

(f) A homomorphism may have an empty kernel.