

Math 361

Quiz 2

10 points possible

1. (5 pts.) Show that if $(ab)^2 = a^2b^2$ in a group G , then $ab = ba$.

2. (5 pts.) Let H be a subgroup of a group G . For $x, y \in G$, define $x \sim y$ to mean $y^{-1}x \in H$. Prove the following.

(a) For all $x \in G$, $x \sim x$.

(b) For all $x, y \in G$, if $x \sim y$, then $y \sim x$.

(c) For all $x, y, z \in G$, if $x \sim y$ and $y \sim z$, then $x \sim z$.

Optional Bonus. (Make as much progress as you can.) Let G be a set with an associative binary operation and an element e such that for all $x \in G$,

(1) $ex = x$, and

(2) there exists $x' \in G$ such that $x'x = e$.

Prove that G is a group. That is, prove that we also have $xe = x$ and $xx' = e$.