MTH 361
Exam 2
Spring 2024

100 points possible. 9 problems at 11 points each, plus 1 free point.

1. Find the order of the given element of the direct product.

$(4, 8, 12)$ in $\mathbb{Z}_9 \times \mathbb{Z}_{10} \times \mathbb{Z}_{18}$

2. What is the largest order among the orders of all the cyclic subgroups of the given group.

(a) $\mathbb{Z}_{18} \times \mathbb{Z}_{30}$

(b) $\mathbb{Z}_6 \times \mathbb{Z}_8 \times \mathbb{Z}_{15}$

3. Disregarding the order of the factors, write direct products of two or more groups of the form $\mathbb{Z}_n$ so that the resulting product is isomorphic to $\mathbb{Z}_{270}$ in as many ways as possible.

4. Find all abelian groups, up to isomorphism, of the given order. Write them in both forms (1) and (2) from the Fundamental Theorem, and pair up isomorphic groups.

As a reminder, in form (1) every subscript is a power of a prime, and in form (2) every subscript divides the next subscript.

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5. An isomorphism of a group with itself is an automorphism of the group. Find the number of automorphisms of the given group.

(a) $\mathbb{Z}_{15}$

(b) $\mathbb{Z}_{18}$

6. (a) Find all cosets of the subgroup $\langle 4 \rangle$ of $\mathbb{Z}_8$. How many distinct cosets did you find?

(b) Find all cosets of the subgroup $\langle 3 \rangle$ of $\mathbb{Z}_{12}$. How many distinct cosets did you find?

7. Find all left cosets of the subgroup $H = \{e, d\}$ of the group G given by this table.

	e	a	b	c	d	f	g	h
e	e	a	b	c	d	f	g	h
a	a	e	c	b	f	d	h	g
b	b	c	d	f	g	h	e	a
c	c	b	f	d	h	g	a	e
d	d	f	g	h	e	a	b	c
f	f	d	h	g	a	e	c	b
g	g	h	e	a	b	c	d	f
h	h	g	a	e	c	b	f	d

(b) Rewrite the above table in the order exhibited by the left cosets in part (a). Do you seem to get a coset group of order 4? If so, is it isomorphic to $\mathbb{Z}_4$ or to the Klein 4-group V?

8. Let G be an abelian group. Prove that the elements of finite order in G form a subgroup.

9. Mark each of the following true or false. (No reasons required.)

(a) Every element in $\mathbb{Z}_9 \times \mathbb{Z}_{27}$ has order 27.

(b) Every subgroup of every group has left cosets.

(c) Every abelian group of prime power order is cyclic.

(d) $\mathbb{Z}_m \times \mathbb{Z}_n$ has mn elements whether m and n are relatively prime or not.

(e) Every finite group contains an element of every order that divides the order of the group.

(f) $\mathbb{Z}_{12}$ is generated by $\{6, 8\}$.