MTH 361
Half Exam 1
Spring 2021

Instructions.

Reminder: intentionally obtaining or attempting to use unauthorized materials or information or unauthorized help from another person is considered cheating.

- Position your webcam to show your hands and work area.
- Keep your hands in view at all times.
- When done with the half exam, type “done” in chat. Stay in the meeting.
- Keep your webcam on, scan and submit on Blackboard, Current Week.
- You may use your phone or iPad to scan after saying you’re “done.”
- You may use your computer to submit on Blackboard or email.
- After submitting, stay in the meeting. Wait for confirmation before leaving.
- Confirmation is a 1 point posted grade, or a confirmation from me in chat.
- If you leave before confirmation: (1) email *and* (2) submit on Blackboard.
- If you leave or turn off your webcam, I have to receive (1) or (2) within 5 minutes.
- Use chat to ask me questions during the exam.

Show work to support each answer. Notes OK but keep hands in view. No book. Graphing calculator OK. No CAS (e.g., no TI-89, no TI-Nspire CAS). No phone, iPad, or other device, except to scan when “done.” Don’t use your computer, except to be in the meeting with your webcam on, to see the half exam problems, and to chat with me to ask any questions. You may use your computer to submit after you’ve scanned.

The problems are on the next page.

50 points possible. (Five problems, 10 points each.)

1. There is an isomorphism of U_{11} with $\mathbb{Z}_{11}$ in which $\zeta = e^{i(2\pi/11)} \leftrightarrow 7$. Find the element in $\mathbb{Z}_{11}$ to which ζ^m must correspond for $m = 0$, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

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2. Determine whether the binary operation $*$ defined is commutative and whether $*$ is associative. Justify.

$$* \text{ defined on } \mathbb{Z} \text{ by letting } a * b = a + 2b$$

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3. The map $\phi : \mathbb{Z} \to \mathbb{Z}$ defined by $\phi(n) = n + 2$ for $n \in \mathbb{Z}$ is one to one and onto $\mathbb{Z}$. Give the definition of a binary operation $*$ on $\mathbb{Z}$ such that ϕ is an isomorphism mapping

$$\langle \mathbb{Z}, \cdot \rangle \text{ onto } \langle \mathbb{Z}, * \rangle$$

Give the identity element for $*$ on $\mathbb{Z}$.

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4. Let G be a group and let b be one fixed element of G. Show that

$$H = \{x \in G \mid bxb^{-1} = x\}$$

is a subgroup of G.

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5. Mark each of the following true or false. (No reasons required.)

(a) Any two groups of three elements are isomorphic.

(b) There may be a group in which the cancellation law fails.

(c) If $*$ is any commutative binary operation on any set S, then $a*(b*c) = (b*c)*a$ for all $a, b, c \in S$.

(d) A group may have more than one identity element.

(e) Every finite group of at most three elements is abelian.

End

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