MTH 361
Quiz 5
Spring 2010

20 points possible.

1. (4 pts.) Let $\langle A, \square \rangle$ and $\langle B, \oplus \rangle$ be binary algebraic structures. Define what it means for ψ to be an **isomorphism** from A to B.

2. (4 pts.) Find all subroups of $\mathbb{Z}_{18}$, and draw the subgroup digram for the subgroups.

3. (4 pts.) Find the *number* of automorphisms of the given group. (You don't have to find the actual automorphisms, just how many there are.)

(a) $\mathbb{Z}_9$

(b) $\mathbb{Z}_{15}$

4. (8 pts.) Determine whether the given map ϕ is an isomorphism of the first binary structure with the second. If so, briefly say why (without formal proof). If not, briefly say why not (i.e., say what fails).

(a) $\langle \mathbb{Q}, \cdot \rangle$ and $\langle \mathbb{Q}, \cdot \rangle$ where $\phi(x) = x^2$ for $x \in \mathbb{Q}$

(b) $\langle \mathbb{Z}, + \rangle$ and $\langle \mathbb{Z}, + \rangle$ where $\phi(n) = 2n$ for $n \in \mathbb{Z}$

(c) $\langle M_2(\mathbb{R}), \cdot \rangle$ and $\langle \mathbb{R}, \cdot \rangle$ where $\phi(A)$ is the determinant of matrix A

(d) $\langle \mathbb{Z}, + \rangle$ and $\langle \mathbb{Z}, + \rangle$ where $\phi(n) = -n$ for $n \in \mathbb{Z}$