Math 361
Exam 1
Spring 2006

100 points possible.

1. (20 pts.) Consider $\tau = (1\,4\,2\,8)(2\,8\,6)(8\,6\,5\,3) \in S_8$.

(a) Write τ in permuation notation.

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ & & & & & & & \end{pmatrix}$$

(b) Write τ as a product of disjoint cycles.

(c) Write τ as a product of transpositions.

(d) Is τ an even or odd permutation?

2. (20 pts.) The set $\mathbb{G} = \{1, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O\}$ and the binary operation $*$ given in the table on the supplemental page form a group (you do not have to prove this).

(a) What is the identity element in $\mathbb{G}$?

(b) Find F^{-1}.

(c) Find $\langle K \rangle$.

(d) Find $|\langle A \rangle|$.

3. (20 pts.) Define a binary operation $*$ on $\mathbb{Z}$ by letting $a * b = ab - 1$ for all $a, b \in \mathbb{Z}$.

(a) Is $*$ commutative? Prove it, or give a numerical counterexample.

(b) Is $*$ associatative? Prove it, or give a numerical counterexample.

4. (20 pts.) Let $n \geq 2$ and define $H = \{\sigma \in S_n \mid (1\,2)\sigma = \sigma(1\,2)\}$. Prove that $H \leq S_n$.

5. (20 pts.) Let $n \geq 3$ and define $\phi : S_n \to S_n$ by

$$\phi(\sigma) = (1\,2\,3)\sigma, \quad \text{for all } \sigma \in S_n.$$

Prove that ϕ is one-to-one and onto. (Recall that $\psi : X \to Y$ is one-to-one iff $\forall x_1, x_2 \in X$, if $\psi(x_1) = \psi(x_2)$, then $x_1 = x_2$. Recall that $\psi : X \to Y$ is onto iff $\forall y \in Y$, $\exists x \in X$ such that $\psi(x) = y$.)