Math 361
Exam 3

Each problem is worth 12.5 points, except for Problem 2, which is worth 25 points.

1. In the construction of the field of quotients F of an integral domain D, prove that the distributive laws hold in F. (You may assume any preceding part of the construction.)

2. Determine whether the given polynomial is reducible or irreducible in the given ring. If it is reducible, factor it completely. If it is irreducible, justify your conclusion.

(a) $x^3 + 2x^2 + x + 3$ in $\mathbb{Z}_5[x]$.

(b) $x^3 + 3x^2 - 15x - 9$ in $\mathbb{Q}[x]$.

(c) $x^4 - 7x^2 + 1$ in $\mathbb{Q}[x]$

(d) $2x^4 - 30x^3 + 60x + 75$ in $\mathbb{Q}[x]$

3. Give the multiplication table for the ring $(\mathbb{Z}_4 \times \mathbb{Z}_2)/N$, where $N = \{(0,0),(2,0)\}$.

4. Find the remainder when 37! is divided by 41.

5. Prove that if I and J are ideals of a ring R, then $I \cap J$ is an ideal of R.

6. Show that $\alpha = \sqrt{5} - i$ is algebraic over $\mathbb{Q}$ by finding a nonzero $f(x) \in \mathbb{Q}[x]$ such that $f(\alpha) = 0$.

7. Find all generators of the cyclic multiplicative group of units of the finite field $\mathbb{Z}_{19}$.