Math 361
Exam 2

1. (a) Define *ring*.

(b) Give an example of a commutative ring with unity $1 \neq 0$ that is not an integral domain.

2. Find the characteristic of the ring $\mathbb{Z}_4 \times \mathbb{Z}_6$.

3. Compute $\operatorname{Ker} \phi$ and $\phi(8)$, where $\phi : \mathbb{Z}_{24} \to \mathbb{Z}_4 \times \mathbb{Z}_3$ is the group homomorphism satisfying $\phi(1) = (2, 2)$.

4. Classify the group $(\mathbb{Z}_4 \times \mathbb{Z}_6)/\langle(2, 2)\rangle$ according to the fundamental theorem of finitely generated abelian groups.

5. Give an example of a nontrivial homomorphism $\phi : \mathbb{Z}_{10} \to \mathbb{Z}_{15}$, if an example exists. If no such homomorphism exists, explain why that is so.

6. List, up to isomorphism, all abelian groups of order 162.

7. Let H and K be subgroups of a group G. Define $\sim$ on G by $a \sim b$ if and only if $a = hbk$ for some $h \in H$ and some $k \in K$. Prove that $\sim$ is an equivalence relation on G. Describe the elements in the equivalence class containing $a \in G$.

8. Prove that $\mathbb{Q}/\mathbb{Z}$ under addition is an infinite group in which every element has finite order.