

MTH 351
Fall 2016

Quiz 1

1. (6 pts.) Alex deposits 60,000 into a bank account that pays an effective annual interest rate of 7% with interest credited at the end of each year. Determine the amount in Alex's account just after interest is credited at the end of the first and second years, and also determine the amount of interest that was credited on each of those dates.

2. (7 pts.) Davina can receive one of the following two payment streams:

(i) 100 at time 0, 300 at time n , and 400 at time $2n$.

(ii) 800 at time 12.

At an effective annual interest rate of i , the present values of the two streams are equal. Given $v^n = 0.64891$, determine i .

3. (7 pts.) For the period from time 0 to time 2, the force of interest is defined as follows:

$$\delta_t = \begin{cases} 0.1 & \text{for } 0 < t \leq 1 \\ 0.1 + 0.04(t - 1) & \text{for } 1 < t \leq 2 \end{cases}$$

1,000 is invested at time 0. Find the accumulated value at time 1 and at time 2.

Quiz 2

1. (6 pts.) A loan of 2000 is repaid with 18 annual payments of 150 each starting one year after the loan is made. The effective annual interest rate is 3.8% for the first 8 years. Find the effective annual interest rate i for the final 10 years.

2. (7 pts.) Sandy purchases a perpetuity-immediate that makes annual payments. The first payment is 300, and each payment thereafter increases by 40. Danny purchases a perpetuity-due which makes annual payments of 650. Using the same annual effective interest rate, $i > 0$, the present value of both perpetuities are equal. Calculate i .

3. (7 pts.) A loan is amortized over six years with monthly payments at a nominal interest rate of 12% compounded monthly. The first payment is 2000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 3% lower than the prior payment. Calculate the outstanding loan balance immediately after the 30th payment is made.

Quiz 3

1. (5 pts.) Invest 8000 in stock now. Four years from now, sell some shares for 7800. Nine years from now, sell the rest for 9500. Assuming an interest rate of 10%, find the net present value.

2. (5 pts.) You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
April 1	15	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y . Calculate Y .

3. (5 pts.) You are given the following table of interest rates (in percents):

y	i_1^y	i_2^y	i_3^y	i_4^y	i^{y+4}	Portfolio Year
1998	7.0	6.5	6.0	5.8	5.9	2002
1999	6.4	6.1	5.8	5.9	6.0	2003
2000	6.2	6.0	5.9	6.0	6.2	2004
2001	6.1	5.9	6.1	6.4	6.6	2005
2002	6.0	6.1	6.3	6.6		
2003	6.4	6.6	6.7			
2004	6.8	7.0				
2005	7.5					

If 100 is invested in the fund at the beginning of 2003, find the accumulated amount at the end of 2005 using:

(a) The investment year method

(b) The portfolio method

(c) The investment year method if the amount is withdrawn at the end of every year and is then reinvested at the new money rate.

4. (5 pts.) Suppose that for the coming year, inflation is forecast at an effective annual rate of $r = 0.18$ and interest is forecast at effective annual rate $i = 0.11$. What will be the corresponding real (i.e., inflation-adjusted) rate of interest for the coming year?

Exam 1

0. (4 pts.) Four free points.

1. (16 pts.) Carly puts 50,000 into a bank account that pays an effective annual interest rate of 6% for eight years, with interest credited at the end of each year. If a withdrawal is made during the first four and one-half years, a penalty of 10% of the withdrawal is made. Carly withdraws K at the end of each of years 4, 5, and 6. The balance in the account at the end of year 8 is 50,000. Calculate K .

2. (16 pts.) Hill Bank pays interest at rate $i^{(2)} = 0.12$. Creek Bank pays interest compounded monthly. What nominal annual rate must Creek Bank pay in order to be equivalent to Hill Bank?

3. (16 pts.) A discounted note of face amount X , due in one-half year, is valued today at 3686. Find X under each of the following interest calculation methods.

(a) Compound interest at effective annual rate 6%.

(b) Simple interest at annual rate 6%.

(c) Compound discount at effective annual rate 6%.

(d) Simple discount at annual rate 6%.

4. (16 pts.) In a series of 30 payments the first 10 payments are 25 each, the second 10 payments are 50 each, and the final 10 payments are 75 each. The payments are equally spaced and the interest rate is 7% per payment period. Find the accumulated value at the time of the final payment.

5. (16 pts.) Robin wants to purchase a perpetuity paying 250 per year with the first payment due at the end of year 7. Robin can purchase it by either:

(i) paying 200 per year at the end of each year for 6 years; or

(ii) paying K per year at the end of each year for the first 3 years and nothing for the next 3 years.

Calculate K .

6. (16 pts.) A perpetuity paying 1 at the beginning of each quarter has a present value of 200. A second perpetuity pays X at the beginning of every 4 years. Assuming the same effective annual interest rate, the two present values are equal. Determine X .

Exam 2

0. (4 pts.) Four free points.

1. (16 pts.) Olga buys a 6-year increasing annuity for X . Olga will receive 5 at the end of the first month, 10 at the end of the second month, and for each month thereafter the payment increases by 5. The nominal interest rate is 8.4% convertible quarterly. Calculate X .

2. (16 pts.) An amortized loan has 5 annual payments at the end of each year starting one year from now. The first 3 payments are \$1000 each and the final 2 payments are \$500 each. Interest is at an effective annual rate of 8%. Find each of the following:

(i) the initial loan amount

(ii) the outstanding balance just after the 2nd payment

(iii) the interest and principal in the 3rd payment

(iv) the outstanding balance just after the 4th payment.

3. (16 pts.) A 20-year loan of 5000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective rate of 6.16%.

(ii) Installments of 250 each year plus interest on the unpaid balance at an annual effective rate of i .

The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine i .

4. (16 pts.) John borrows 1000 for 15 years at an annual effective interest rate of 9%. He can repay this loan using the amortization method with payments of P at the end of each year. Instead, John repays the 1000 using a sinking fund that pays an annual effective rate of 12%. The deposits to the sinking fund are equal to P minus the interest on the loan and are made at the end of each year for 15 years. Determine the balance in the sinking fund immediately after repayment of the loan.

5. (16 pts.) Among a company's assets and accounting records, an actuary finds a 20-year bond that was purchased at a premium. From the records, the actuary has determined the following:

(i) The bond pays semi-annual interest.

(ii) The amount for amortization of the premium in the 3rd coupon payment was 429.32.

(iii) The amount for amortization of the premium in the 5th coupon payment was 466.14.

What is the value of the premium?

6. (16 pts.) An 8% bond with face amount 100 is callable on any coupon date from $25\frac{1}{2}$ years after issue up to the maturity date which is 30 years from issue. Assume the bond is callable at a redemption amount of 115, including the redemption at maturity.

(a) Find the price of the bond to yield a minimum nominal annual rate of 10%.

(b) Find the price of the bond to yield a minimum nominal annual rate of 8%.

(c) Find the price of the bond to yield a minimum nominal annual rate of 6%.