MTH 301
Exam 1
Spring 2020

100 points possible.

0. (1 pt.) One free point.

1. (11 pts.) Use a truth table to determine whether the argument form is valid or invalid. Indicate which columns represent the premises and which represents the conclusion of the argument. Clearly label the "critical rows." Don't forget to say "valid" or "invalid"!

$$\begin{array}{ll} & p \lor q \\ & p \to \sim q \\ & p \to r \\ \therefore & r \end{array}$$

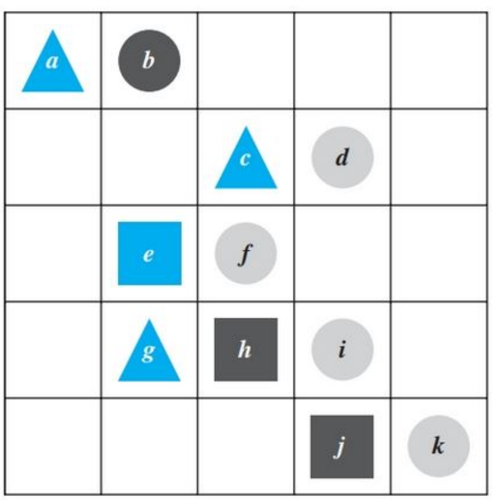
2. (11 pts.) A supplemental page shows a circuit (labeled "Problem 2").

Write the input/output table for the circuit.

P	Q	R	S

3. (11 pts.) Consider the Tarski world figure below. Indicate whether each statement is true or false. (No justification needed. Write the whole word "true" or "false.")

Reminders: Triangle(x) means "x is a triangle." Square(x) means "x is a square." Above(x, y) means "x is above y (but not necessarily in the same column)"; i.e., "the row with x is above the row with y."



(a) $\exists y$ such that Square(y) $\wedge$ Above(y, d)

(b) $\forall u$, Triangle(u) $\rightarrow$ Above(u, f)

(c) For all squares w, there is a triangle y such that y is above w.

4. (11 pts.) Write the negation each statement.

(a) $\exists$ a movie x such that x is over 4 hours long.

(b) $\forall x \in \mathbb{R}$, if $x(x+3) > 0$, then $x < -3$ or $x > 0$.

(c) $\exists$ a course y such that $\forall$ student z, z has taken y.

5. (11 pts.) State whether each argument is valid or invalid. (No justification needed.)

(a) All overconfident people impress the dean.

Burke impresses the dean.

$\therefore$ Burke is an overconfident person.

(b) If a subring has an identity element, then it has the commutative property.

This subring does not have an identity element.

$\therefore$ This subring does not have the commutative property.

(c) All directors accumulate clutter.

No candidates ever accumulate clutter.

$\therefore$ No directors are candidates.

6. (11 pts.) Fill in the blanks in the following proof that the square of any rational number is rational. (Note: Some blanks might be filled by a single variable, and others by a short phrase.)

Theorem. *For every rational number r, r^2 is rational.*

Proof. Suppose that r is ___(a)___. By definition of rational, $r = a/b$ for some ___(b)___ with $b \neq 0$. By substitution,

$$r^2 = \underline{\quad (c) \quad} = a^2/b^2.$$

Since a and b are both integers, so are the products a^2 and ___(d)___. Also $b^2 \neq 0$ by the ___(e)___. Hence r^2 is a ratio of two integers with a non-zero denominator, and so ___(f)___ by definition of rational. □

(a)

(b)

(c)

(d)

(e)

(f)

7. (11 pts.) Prove. (Prove directly from the definitions.)

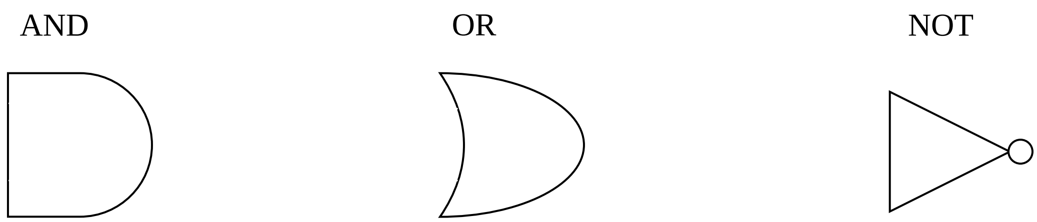
For all integers a, b, and c, if $a \mid b$ and $a \mid c$ then $a \mid (4b - c)$.

8. (11 pts.) Prove. (Use the quotient-remainder theorem with divisor equal to 3.)

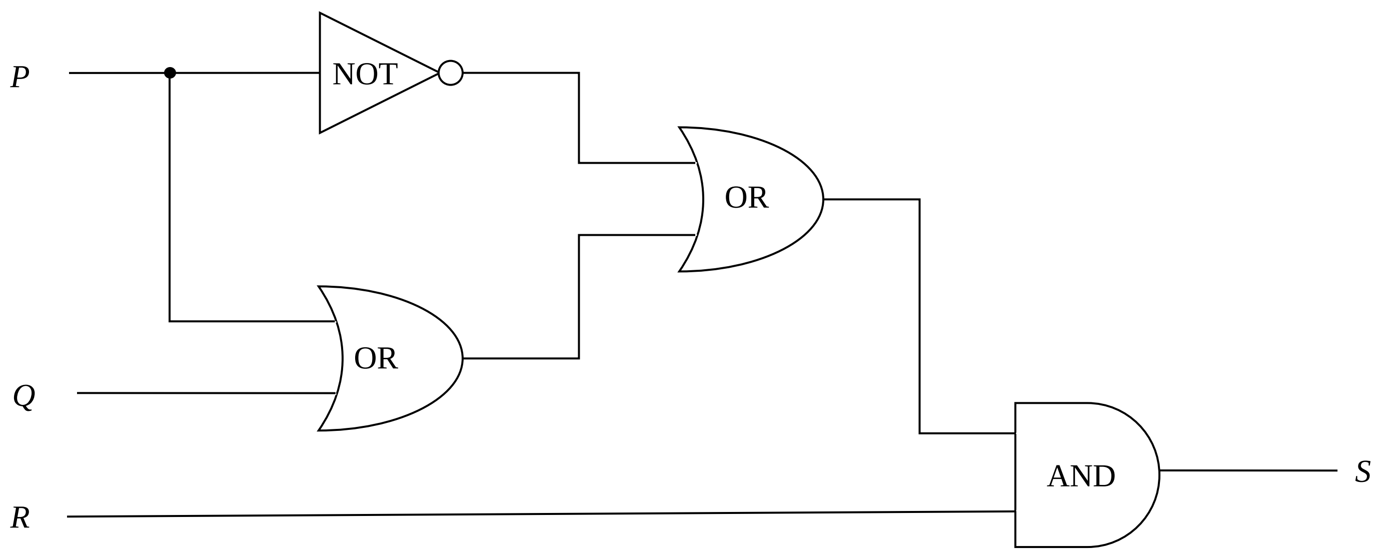
The square of any integer has the form $3k$ or $3k + 1$ for some integer k.

9. (11 pts.) Find the final values of j, s, and t after the following algorithm is executed. A supplemental page has the corresponding flowchart.

$j := -2$
$s := 8$
$t := 30$
while $j \neq 3$
  if ($j < 0$ or $j = 2$)
    then $t := t + j$
    else $s := s + 5$
  $j := j + 1$
end while



Problem 2.



Problem 9.