

MTH 301
Quiz 2
Spring 2018

20 points possible.

1. (6 pts.) Let $A = \{5, 6, 7\}$ and $B = \{3, 6, 9\}$.

(a) Define a relation R from A to B as follows. For all $(x, y) \in A \times B$,

$$(x, y) \in R \text{ means that } x \leq y.$$

(i) Draw the arrow diagram for R .

(ii) Indicate whether or not R is a function from A to B .

(b) Define a relation T from A to B as follows.

$$T = \{(5, 3), (7, 6), (7, 9)\}$$

(i) Draw the arrow diagram for T .

(ii) Indicate whether or not T is a function from A to B .

2. (4 pts.) Yes or no.

(a) Is $\{3\} \in \{1, 2, 3\}$?

(b) Is $\{2\} \in \{1, \{2\}, \{3\}\}$?

(c) Is $\{1\} \subseteq \{1, 2, 3\}$?

(d) Is $3 \in \{1, \{2\}, \{3\}\}$?

3. (5 pts.) Here is a proof of a theorem. Fill in the blanks. (Note: Some blanks might be filled by a single variable, others by a short phrase, and others by one or more sentences.)

Theorem. For any sets A , B , and C , $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

Proof. Suppose A , B , and C are any sets.

Let $x \in A \cap (B \cup C)$. [We must show that $x \in$ (a) .] By definition of intersection, $x \in$ (b) and $x \in$ (c) . Thus, $x \in A$ and by definition of union, $x \in B$ or (d) .

Case 1 ($x \in A$ and $x \in B$): In this case, by definition of intersection $x \in$ (e) , and so by definition of union, $x \in (A \cap B) \cup (A \cap C)$.

Case 2 ($x \in A$ and $x \in C$): In this case, (f) .

Hence in either case, $x \in (A \cap B) \cup (A \cap C)$ [as was to be shown].

[So $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ by definition of subset.] \square

(a)

(b)

(c)

(d)

(e)

(f) (Longer response)

4. (5 pts.) Prove using mathematical induction.

For all integers $n \geq 1$,

$$5 + 11 + 17 + \cdots + (6n - 1) = n(3n + 2).$$