**MTH 301**
**Exam 1**
**Spring 2018**

100 points possible.

**1.** (8 pts.) Write the truth table for the following statement form.

$$(p \vee \sim q) \wedge \sim r$$

**2.** (12 pts.) Write the negation for each statement.
(Assume each variable represents a fixed entity.)

(a) CL is a math major and Suho is a computer science major.

(b) If this program contains an error, then DATAENDFLAG is off.

(c) If $n$ is prime, then $n$ is odd or $n$ is 8.

**3.** (8 pts.) Write the contrapositive for each statement.
(Assume each variable represents a fixed entity.)

(a) If $P$ is a hexagon, then $P$ is a polygon.

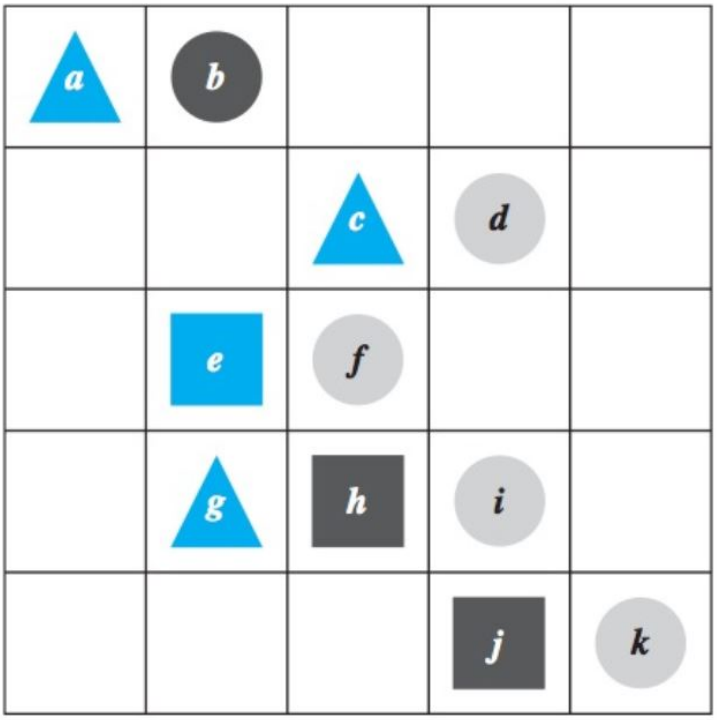
(b) If $k$ is an integer, then $k$ is a real number and $k$ is a rational number.

**4.** (8 pts.) For the given truth table, construct a Boolean expression having the given table as its truth table. (The inputs are $P$, $Q$, and $R$; the output is $S$.)

| $P$ | $Q$ | $R$ | $S$ |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

**5.** (16 pts.) Consider the Tarski world figure below. Indicate whether each statement is true or false. (No justification needed. Write the whole word "true" or "false.")

Reminders: Circle($x$) means "$x$ is a circle." Above($x, y$) means "$x$ is above $y$ (but not necessarily in the same column)"; i.e., "the row with $x$ is above the row with $y$."




(a) $\exists y$ such that Circle($y$) $\wedge$ Above($y, d$)

(b) $\forall u,$ Circle($u$) $\rightarrow$ Above($u, f$)

(c) There is a square $z$ such that for all triangles $v$, $v$ is above $z$.

(d) For all circles $w$, there is a square $y$ such that $w$ is above $y$.

**6.** (16 pts.) State whether each argument is valid or invalid. (No justification needed.)

(a) If Lord Merton used the correct procedure, then Lord Merton obtained the answer 3.14.

Lord Merton did not use the correct procedure.

$\therefore$ Lord Merton did not obtain the answer 3.14.

(b) All healthy people eat an apple a day.

Isobel is not a healthy person.

$\therefore$ Isobel does not eat an apple a day.

(c) All healthy people eat an apple a day.

Carson is a healthy person.

$\therefore$ Carson eats an apple a day.

(d) All staff members eat an apple a day.

No CS majors eat an apple a day.

$\therefore$ No staff members are CS majors.

**7.** (8 pts.) Compute the summation or product.

(a) $\sum_{k=1}^{4}(k+2)$

(b) $\prod_{k=2}^{3} k^2$

**8.** (8 pts.) Fill in the blanks in the following proof by contradiction that there is no least positive number. (Note: Some blanks might be filled by a single variable, and others by a short phrase.)

**Theorem.** *There is no least positive number.*

*Proof.* Suppose not. That is, suppose that there is a least positive real number $x$. *[We must deduce* __(a)__ *.]* Consider the number $x/2$. Since $x$ is a positive real number, $x/2$ is also __(b)__. In addition, we can deduce that $x/2 < x$ by multiplying both sides of the inequality $1 < 2$ by __(c)__ and dividing by __(d)__. Hence $x/2$ is a positive real number that is less than the least positive real number. This is a __(e)__. *[Thus the supposition is false, and so there is no least positive real number.]* □

(a)

(b)

(c)

(d)

(e)

**9.** (8 pts.) Prove. (Prove directly from the definitions.)

The sum of any two even integers is an even integer.

**10.** (8 pts.) Prove. (Prove directly from the definitions.)

For all integers $a$, $b$, and $c$, if $a \mid b$ and $a \mid c$ then $a \mid (5b + c)$.