MTH 301
Exam 2
Spring 2013

100 points possible.

1. (6 pts.) Define *rational number*.

2. (12 pts.) Write the negation for each of the following statements.

(Move the negation "all the way inward," or "all the way right," just as we always did in class.)

(a) For all $x \in D$, if $3x + 8 > 0$, then $x^3 > 5$ or $9x \in S$.

(b) $\forall$ even integers n, $\exists w \in T$ such that $n^2 + 5w < -1$.

3. (6 pts.) Write the contrapositive for the following statement.
$\forall$ positive real numbers t, if $t^2 - 6 \in B$ or $|t + 20| \leq 80$, then $3t \in C$.

4. (16 pts.) Decide whether each argument is valid or invalid.

(a) Any sum of two isonary numbers is isonary.

The sum $a + b$ is isonary.

$\therefore$ The numbers a and b are both isonary.

(b) All diffeostatic manifolds are hydromorphic.

G is not a diffeostatic manifold.

$\therefore$ G is not hydromorphic.

(c) No good cars are cheap.

A Plastodrive is not a good car.

$\therefore$ A Plastodrive is cheap.

(d) No college cafeteria food is wasted.

All food made with MSG is wasted.

$\therefore$ No college cafeteria food is made with MSG.

5. (12 pts.) Prove the following statements.

(a) There are distinct integers m and n such that $\frac{1}{m} + \frac{1}{n}$ is an integer.

(b) There is an integer $n > 3$ such that $2^n - 1$ is prime.

6. (12 pts.) Prove the following statement.

The sum of any two odd integers is even.

7. (12 pts.) Prove the following statement.

For all integers a, b, and c, if $a \mid b$ and $a \mid 2c$, then $a \mid (11b - 6c)$.

8. (12 pts.) Prove the following statement.

For all integers n, $3n^2$ is of the form $9k$ or $9k + 3$ for some integer k.

(Hint: Divide n by 3.)

9. (12 pts.) Find the final values of j, s, t, a, and b after the following algorithm is executed. A supplemental page has the corresponding flowchart.

$j := 0$
$s := 3$
$t := 8$
$a := 10$
$b := 5$
while $j \leq 2$
    if ($j < 1$ or $j = 2$)
        then $t := t + j$
        else $s := s + 2$
    $a := a + 10$
    $b := b + a$
    $j := j + 1$
end while