MTH 301
Exam 1
Spring 2013

100 points possible.

1. (4 pts.) Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentence in symbolic notation.

DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.

2. (8 pts.) Construct a complete truth table for both of these statement forms. Then state whether or not the two statement forms are logically equivalent, briefly justifying your answer.

$$(p \vee q) \wedge r \qquad \text{and} \qquad p \vee (q \wedge r)$$

3. (8 pts.) Write the negation for each of the following statements. (Assume x is a particular real number.)

(a) Fox is investigating the loss of time or Dana is submitting a report.

(b) $-5 < x < 10$

4. (8 pts.) A supplemental page shows a circuit (labeled "Problem 4").

(a) Write an input/output table for the circuit.

(b) Write the Boolean expression that corresponds to the circuit.

5. (8 pts.) For the given truth table, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table. (The inputs are P, Q, and R; the output is S. The symbolic representations of the NOT-, AND-, and OR-gates are shown on a supplemental page.)

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

6. (4 pts.) A supplemental page shows a circuit (labeled "Problem 6"); this circuit uses two half-adders. (The given circuit is *not* a full-adder.)

Give the output signals S and T if the input signals P, Q, and R are as specified.

Also give the intermediate signals C_1, S_1, C_2, and S_2.

$P = 1$, $Q = 1$, $R = 1$

7. (8 pts.) Write the negation for each of the following statements.

(a) If Minzy is clapping her hands, then Bom is going away.

(b) If G-Dragon is telling me goodbye, then Taeyang is a liar and Daesung does not have wings.

8. (8 pts.) Write the contrapositive for each of the following statements.

(a) If Anne is miserable, then Sally is in the field.

(b) If Joe gives little things away and Mike holds his hands high, then Chester leaves out all the rest.

9. (4 pts.) Rewrite this statement in if-then form.

Pete's being a bootless half-faced vassal is a sufficient condition for Stu to be a dissembling clay-brained puttock.

10. (8 pts.) Use a truth table to determine whether the argument form is valid or invalid. Clearly label the "critical rows."

$p \rightarrow \sim q$
$\sim r \rightarrow \sim p$
$\therefore p \vee q$

11. (12 pts.) Each argument exhibits modus ponens, modus tollens, the converse error, or the inverse error. State whether each argument is valid or invalid, and state whether the form is modus ponens, modus tollens, the converse error, or the inverse error.

(a) If you are spacious in the possession of dirt, then you are hidden from this open and apparent shame.

You are not spacious in the possession of dirt.

$\therefore$ You are not hidden from this open and apparent shame.

(b) If Brick whispers to himself, then Sue's middle name is Sue.

Sue's middle name is Sue.

$\therefore$ Brick whispers to himself.

(c) If a Road Block is a task that only one person may perform, then a Detour is a choice between two tasks.

A Detour is not a choice between two tasks.

$\therefore$ A Road Block is not a task that only one person may perform.

12. (8 pts.) Let $P(x)$ be the predicate "$x < 1/x$."

(a) Write $P(3)$, $P(-3)$, $P(\frac{1}{3})$, and $P(-\frac{1}{3})$, and indicate which of these statements are true and which are false.

(b) Find the truth set of $P(x)$ if the domain of x is $\mathbb{R}$, the set of all real numbers.

13. (4 pts.) Rewrite the following statement in the two forms "$\forall x$, if _____ then _____" and "$\forall$_____x, _____"

All singers are songwriters.

14. (8 pts.) For the Tarski World shown on a supplemental page, determine whether each statement is true or false. Below(x, y) means x is closer to the bottom than y.

(a) $\forall u,\ \text{Square}(u) \rightarrow \text{Below}(u, c)$

(b) $\exists u$ such that $\text{Circle}(u) \wedge \text{Below}(u, e)$