MTH 301
Spring 2013
Exam 1

1. (4 pts.) Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentence in symbolic notation.

DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.

2. (8 pts.) Construct a complete truth table for both of these statement forms. Then state whether or not the two statement forms are logically equivalent, briefly justifying your answer.

$$(p \vee q) \wedge r \qquad \text{and} \qquad p \vee (q \wedge r)$$

3. (8 pts.) Write the negation for each of the following statements. (Assume x is a particular real number.)

(a) Fox is investigating the loss of time or Dana is submitting a report.

(b) $-5 < x < 10$

4. (8 pts.) A supplemental page shows a circuit (labeled "Problem 4").

(a) Write an input/output table for the circuit.

(b) Write the Boolean expression that corresponds to the circuit.

5. (8 pts.) For the given truth table, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table. (The inputs are P, Q, and R; the output is S. The symbolic representations of the NOT-, AND-, and OR-gates are shown on a supplemental page.)

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

6. (4 pts.) A supplemental page shows a circuit (labeled "Problem 6"); this circuit uses two half-adders. (The given circuit is *not* a full-adder.)

Give the output signals S and T if the input signals P, Q, and R are as specified.

Also give the intermediate signals C_1, S_1, C_2, and S_2.

$P = 1$, $Q = 1$, $R = 1$

7. (8 pts.) Write the negation for each of the following statements.

(a) If Minzy is clapping her hands, then Bom is going away.

(b) If G-Dragon is telling me goodbye, then Taeyang is a liar and Daesung does not have wings.

8. (8 pts.) Write the contrapositive for each of the following statements.

(a) If Anne is miserable, then Sally is in the field.

(b) If Joe gives little things away and Mike holds his hands high, then Chester leaves out all the rest.

9. (4 pts.) Rewrite this statement in if-then form.

Pete's being a bootless half-faced vassal is a sufficient condition for Stu to be a dissembling clay-brained puttock.

10. (8 pts.) Use a truth table to determine whether the argument form is valid or invalid. Clearly label the "critical rows."

$p \to \sim q$

$\sim r \to \sim p$

$\therefore p \vee q$

11. (12 pts.) Each argument exhibits modus ponens, modus tollens, the converse error, or the inverse error. State whether each argument is valid or invalid, and state whether the form is modus ponens, modus tollens, the converse error, or the inverse error.

(a) If you are spacious in the possession of dirt, then you are hidden from this open and apparent shame.
You are not spacious in the possession of dirt.
$\therefore$ You are not hidden from this open and apparent shame.

(b) If Brick whispers to himself, then Sue's middle name is Sue.
Sue's middle name is Sue.
$\therefore$ Brick whispers to himself.

(c) If a Road Block is a task that only one person may perform, then a Detour is a choice between two tasks.
A Detour is not a choice between two tasks.
$\therefore$ A Road Block is not a task that only one person may perform.

12. (8 pts.) Let $P(x)$ be the predicate "$x < 1/x$."

(a) Write $P(3)$, $P(-3)$, $P(\frac{1}{3})$, and $P(-\frac{1}{3})$, and indicate which of these statements are true and which are false.

(b) Find the truth set of $P(x)$ if the domain of x is $\mathbb{R}$, the set of all real numbers.

13. (4 pts.) Rewrite the following statement in the two forms
"$\forall x$, if ___________ then ___________" and "$\forall$___________ x, ___________"
All singers are songwriters.

14. (8 pts.) For the Tarski World shown on a supplemental page, determine whether each statement is true or false. Below(x, y) means x is closer to the bottom than y.

(a) $\forall u,\ \text{Square}(u) \to \text{Below}(u, c)$

(b) $\exists u$ such that $\text{Circle}(u) \wedge \text{Below}(u, e)$

Exam 2

1. (6 pts.) Define *rational number*.

2. (12 pts.) Write the negation for each of the following statements.
(Move the negation "all the way inward," or "all the way right," just as we always did in class.)

(a) For all $x \in D$, if $3x + 8 > 0$, then $x^3 > 5$ or $9x \in S$.

(b) $\forall$ even integers n, $\exists w \in T$ such that $n^2 + 5w < -1$.

3. (6 pts.) Write the contrapositive for the following statement.
$\forall$ positive real numbers t, if $t^2 - 6 \in B$ or $|t + 20| \leq 80$, then $3t \in C$.

4. (16 pts.) Decide whether each argument is valid or invalid.

(a) Any sum of two isonary numbers is isonary.
The sum $a + b$ is isonary.
$\therefore$ The numbers a and b are both isonary.

(b) All diffeostatic manifolds are hydromorphic.
G is not a diffeostatic manifold.
$\therefore$ G is not hydromorphic.

(c) No good cars are cheap.
A Plastodrive is not a good car.
$\therefore$ A Plastodrive is cheap.

(d) No college cafeteria food is wasted.
All food made with MSG is wasted.
$\therefore$ No college cafeteria food is made with MSG.

5. (12 pts.) Prove the following statements.

(a) There are distinct integers m and n such that $\frac{1}{m} + \frac{1}{n}$ is an integer.

(b) There is an integer $n > 3$ such that $2^n - 1$ is prime.

6. (12 pts.) Prove the following statement.

The sum of any two odd integers is even.

7. (12 pts.) Prove the following statement.

For all integers a, b, and c, if $a \mid b$ and $a \mid 2c$, then $a \mid (11b - 6c)$.

8. (12 pts.) Prove the following statement.

For all integers n, $3n^2$ is of the form $9k$ or $9k + 3$ for some integer k.

(Hint: Divide n by 3.)

9. (12 pts.) Find the final values of j, s, t, a, and b after the following algorithm is executed. A supplemental page has the corresponding flowchart.

```
j := 0
s := 3
t := 8
a := 10
b := 5
while j ≤ 2
    if (j < 1 or j = 2)
        then t := t + j
        else s := s + 2
    a := a + 10
    b := b + a
    j := j + 1
end while
```

Exam 3

1. (10 pts.) The binary search algorithm and flowchart are on supplemental pages. Perform a binary search trace on 3, 6, 7, 10, 11, 12, 14, 15. Search for 12. Show your work in a trace table.

2. (20 pts.) Continually subtract, in turn, the less from the greater, and take note of the last nonzero remainder. Show your work.

(a) 307 and 617

(b) 624 and 390

3. (10 pts.) Find the least positive inverse for 25 modulo 67. Show your work.

4. (10 pts.) Find 21^{13} mod 23. Show your work.

5. (10 pts.) Compute the summation.

$$\sum_{m=0}^{3} \frac{1}{2^m}$$

6. (10 pts.) Write using summation notation.

$$\frac{3}{4 \cdot 5} - \frac{4}{5 \cdot 6} + \frac{5}{6 \cdot 7} - \frac{6}{7 \cdot 8} + \frac{7}{8 \cdot 9}$$

7. (15 pts.) Prove.

For all integers a, b, and c, if $a \nmid (9b - 10c)$, then $a \nmid 3b$ or $a \nmid 5c$.

8. (15 pts.) Prove using the Principle of Mathematical Induction.

$$\sum_{i=1}^{n} (7i - 5) = \frac{n(7n - 3)}{2} \qquad \text{for all integers } n \geq 1$$

Exam 4

1. (6 pts.) Define **relation** from a set E to a set T.

2. (6 pts.) Let $f : A \to B$. Define what it means for f to be **one-to-one**.

3. (6 pts.) Let $f : A \to B$. Define what it means for f to be **onto**.

4. (6 pts.) Let R be a relation on a set A. Define what it means for R to be **transitive**.

5. (8 pts.) Let the universal set be the set $\mathbf{R}$ of all real numbers. Also let $A = \{x \in \mathbf{R} \mid 0 < x \leq 5\}$, $B = \{x \in \mathbf{R} \mid 2 \leq x < 8\}$, and $C = \{x \in \mathbf{R} \mid 6 \leq x < 10\}$. Find each of the following.

(a) $A \cap B$

(b) $A \cup C$

(c) $B^c \cap C^c$

6. (8 pts.) Let $V_i = \{x \in \mathbf{R} \mid 0 \leq x \leq i\} = [0, i]$ for all positive integers i.

(a) Find $\displaystyle\bigcup_{i=1}^{4} V_i$.

(b) Find $\displaystyle\bigcap_{i=1}^{n} V_i$.

(c) Find $\displaystyle\bigcup_{i=1}^{\infty} V_i$.

7. (8 pts.) Let S be the set of all strings of a's, b's, and c's.
Define $F : S \to \mathbf{Z}$ by letting $F(s) =$ the number of c's in s, for all $s \in S$.

(a) Is F one-to-one? Briefly explain your reasoning, or give a counterexample.

(b) Is F onto? Briefly explain your reasoning, or give a counterexample.

8. (8 pts.) Let $A = \{0, 1, 2, 3\}$ and let R be defined on A as follows.

$$R = \{(0, 1), (0, 3), (1, 0)\}$$

Find the transitive closure of R.

9. (8 pts.) Let $A = \{i \in \mathbf{Z} \mid -9 \leq i \leq 9\} = \{-9, -8, -7, \ldots, 7, 8, 9\}$. Let R be defined on A as follows.

$$\text{For all } x, y \in A, \quad x \; R \; y \quad \Leftrightarrow \quad 3 \mid (x - y)$$

It can be shown R is an equivalence relation on A (you don't have to prove it).
Find the distinct equivalence classes of R.

10. (12 pts.) Prove by mathematical induction.
$7^n - 1$ is divisible by 3, for each integer $n \geq 1$.

11. (12 pts.) Suppose that $b_0, b_1, b_2, \ldots$ is a sequence defined as follows:

$$b_0 = 3, \; b_1 = 10,$$
$$b_k = 7b_{k-1} - 12b_{k-2} \quad \text{for all integers } k \geq 2.$$

Prove, using strong induction, that $b_n = 4^n + 2 \cdot 3^n$ for all integers $n \geq 0$.

12. (12 pts.) Here is a proof of a theorem. Fill in the blanks. (Note: Some blanks might be filled by a single variable, others by a short phrase, and others by one or more sentences.)

Theorem. *For any sets A, B, and C,* $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

Proof. Suppose A, B, and C are particular but arbitrarily chosen sets.
Let $x \in A \cap (B \cup C)$. *[We must show that* $x \in$ ___(a)___*.]* By definition of intersection, $x \in$ ___(b)___ and $x \in$ ___(c)___. Thus, $x \in A$ and by definition of union, $x \in B$ or ___(d)___.

Case 1 ($x \in B$): In this case, since we know $x \in A$, then by definition of intersection $x \in$ ___(e)___, and so by definition of union, $x \in (A \cap B) \cup (A \cap C)$.

Case 2 ($x \notin B$): In this case, we must have $x \in C$. Since we know $x \in A$, then ___(f)___.

Hence in either case, $x \in (A \cap B) \cup (A \cap C)$ *[as was to be shown].*

[So $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ *by definition of subset.]* ☐