

Math 301
Quiz 3
Spring 2005

Be neat and organized. Clearly indicate your answers. 20 points possible.

1. (5 pts.) The Zero Product Property says that if a product of two real numbers is 0, then one of the numbers must be 0.

(a) Write this property formally using quantifiers (\forall and/or \exists) and variables.

(b) Write the contrapositive of your answer to part (a).

2. (5 pts.) Fill in the blanks in the proof.

Theorem. *The sum of any two even integers is even.*

Proof. Suppose k and w are particular but arbitrarily chosen (a). [We must show that (b).] By definition of even, $k = 2n$ and $w = 2m$ for some (c). By substitution and algebra, $k + w =$ (d) $= 2(n + m)$. Let $j = n + m$. Note that j is an integer because (e). Hence $k + w = 2j$ where j is an integer. It follows by definition of even that $k + w$ is even. ■

(a)

(b)

(c)

(d)

(e)

3. (5 pts.) Prove the given statement directly from the definitions.

The sum of any integer and any rational number is a rational number.

4. (5 pts.) Prove the given statement directly from the definitions.

For all integers b , k , and n , if $b \mid k$ and $k \mid n$, then $b \mid (8k - 5n)$.