

Math 301
Exam 2
Spring 2005

Be neat and organized. Clearly indicate your answers. 100 points possible.

1. (10 pts.) Let the universal set be the set \mathbf{R} of all real numbers. Also let $A = \{x \in \mathbf{R} \mid 0 < x \leq 5\}$ and $B = \{x \in \mathbf{R} \mid 2 \leq x < 8\}$. Find each of the following.

(a) $A \cup B$

(b) $A \cap B$

2. (10 pts.) Let $A = \{4, 7\}$.

(a) Find $\mathcal{P}(A)$ (the power set of A). Also, how many elements are in $\mathcal{P}(A)$?

(b) Find $A \times A$. Also, how many elements are in $A \times A$?

3. (10 pts.) A pair of ordinary dice is rolled. Find the probability that the sum of the numbers showing face up is 9.

4. (10 pts.) There are two labeled jars: Jar A contains 4 red marbles and 3 white marbles, while Jar B contains 2 red marbles and 7 white marbles.

An ordinary die is tossed. If the number showing face up is a 6, then a marble is selected at random from Jar A. If the number showing face up is not a 6, then a marble is selected at random from Jar B.

Find the probability that the selected marble is red.

5. (15 pts.) Prove the following statement. (Prove it directly from the definitions.)

The sum of any two odd integers is even.

6. (15 pts.) Prove the following statement. (Prove it directly from the definitions.)

For all integers a , w , and k , if $w \mid a$ and $w \mid k$, then $w \mid (5a - 9k)$.

7. (15 pts.) Prove the following statement by contraposition.

For all integers b , if b^3 is odd, then b is odd.

8. (15 pts.) Use the Principle of Mathematical Induction to prove the following statement.

$$2 + 7 + 12 + \cdots + (5n - 3) = \frac{n(5n - 1)}{2}, \quad \text{for all integers } n \geq 1.$$