Math 301
Exam 1
Spring 2005

Read the instructions and write your answers carefully. Missing a "little" word like "and," "or," "not," or "if" could result in no credit for that problem. Don't write trivial negations, like "It is not the case that *blah blah*." Show me you have learned something.

Be neat and organized. Clearly indicate your answers. 100 points possible.

1. (10 pts.) Determine if the two statement forms are logically equivalent. Justify your answer using a complete truth table.

$$(p \wedge q) \vee r$$

$$p \wedge (q \vee r)$$

2. (10 pts.) Write negations for each of the following statements.

In part (c), assume x is a particular real number.

(a) George is tall and Ringo is short.

(b) Carole took the high road or Carly took the low road.

(c) $2 < x \leq 8$

3. (10 pts.) Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Justify your answer with a complete truth table.

If she is here and she is there, then she is everywhere.

Either she is not here or she is not there, or she is everywhere.

If she is not here and she is not there, then she is not everywhere.

4. (10 pts.) Use the contrapositive to rewrite each statement in if-then form in two ways.

In part (a), assume that "only if" has its formal, logical meaning.

(a) Moneybags will let Spryo go only if Sheila has rescued Sgt. Byrd.

(b) Being able to turn off my mind is a necessary condition for my being able to float downstream.

5. (10 pts.) Each argument exhibits modus ponens, modus tollens, the converse error, or the inverse error. Decide whether each argument is valid or invalid, and state whether the form is modus ponens, modus tollens, the converse error, or the inverse error.

(a) If this Katamari is at least 10 cm in diameter, then it can pick up a pencil.

This Katamari can not pick up a pencil.

$\therefore$ This Katamari is not at least 10 cm in diameter.

(b) If this graph has an Euler circuit, then every vertex of this graph has even degree.

This graph has an Euler circuit.

$\therefore$ Every vertex of this graph has even degree.

(c) If Judit finds the right move, then Peter will resign.

Peter did not resign.

$\therefore$ Judit did not find the right move.

6. (5 pts.) Let D be the set of all students at Elmhurst College. Let $M(s)$ be "s is a math major," let $C(s)$ be "s is a computer science student," and let $P(s)$ be "s is a physics student." Express each statement using the symbol $\forall$ or $\exists$, variables, and the predicates $M(s)$, $C(s)$, or $P(s)$.

(a) There is a physics student who is a computer science student.

(b) Every math major is a physics student.

7. (5 pts.) Consider the following statement.

$(**)$ $\forall$ characters x, if x's father is a Texan, then x is a Texan.

For each statement, decide whether or not it is equivalent to statement $(**)$.

(a) All characters have fathers who are Texans and are Texans themselves.

(b) There exists a character x such that x's father is a Texan but x is not a Texan.

(c) Any character with a Texan for a father is a Texan.

(d) If the father of a character is a Texan, then that character is a Texan.

8. (10 pts.) Consider the following statement.

$(*)$ $\forall$ Magic-users w, if w is a Theurgist or w is a Necromancer, then w is a Prestidigitator.

(a) Write the contrapositive of statement $(*)$.

(b) Write the converse of statement $(*)$.

(c) Write the inverse of statement $(*)$.

9. (10 pts.) A college cafeteria line has four stations: salads, main courses, desserts, and beverages. The salad station offers a choice of green salad or fruit salad; the main course station offers spaghetti or fish; the dessert station offers pie or cake; the beverage station offers milk, soda, or coffee. Three students, Uta, Tim, and Yuen, go through the line and make the following choices:

Uta: green salad, spaghetti, pie, milk

Tim: fruit salad, fish, pie, cake, milk, coffee

Yuen: spaghetti, fish, pie, soda

Determine whether each of the following statements is true or false.

(a) $\exists$ a student S such that $\forall$ items I, S chose I.

(b) $\exists$ a student S such that $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

(c) $\forall$ students S, $\exists$ a salad T such that S chose T.

10. (10 pts.) Each statement refers to a Tarski world in formal, logical notation. Write the negation in formal, logical notation.

(a) $\exists x(\text{Square}(x) \wedge (\forall y(\text{Triangle}(y) \rightarrow \text{Above}(x, y))))$

(b) $\forall x(\text{Triangle}(x) \rightarrow (\exists y(\text{Circle}(y) \wedge \text{RightOf}(x, y))))$

11. (10 pts.) Indicate whether the arguments are valid or invalid. You may draw a diagram to support your answers, if you wish.

(a) All people are mice.

All mice are mortal.

$\therefore$ All people are mortal.

(b) All intelligent investors take the trouble to look beyond current yields in evaluating short-term bond funds.

No foolish people are intelligent investors.

$\therefore$ No foolish people take the trouble to look beyond current yields in evaluating short-term bond funds.

(c) No bleeps are zips.

No zips are globs.

$\therefore$ No bleeps are globs.