Math 301
Quiz 11

10 points possible.

1. (5 pts.) Recall that the ordinary Tower of Hanoi is a game consisting of n circular rings of varying size and three pegs on which the rings fit. Initially the rings are placed on the first (left-most) peg with the largest ring at the bottom covered by successively smaller rings. By transferring the rings among the pegs, one seeks to achieve a similarly tapered pile on the third (right-most) peg. Rings may be moved only one at a time. Each time a ring is transferred to a new peg, the transferred ring must be smaller than any of the rings already piled on this new peg.

For each integer $n \geq 1$, let m_n be the minimum number of moves needed to move a tower of n disks from the first peg to the third peg in the ordinary Tower of Hanoi game.

(a) Find m_4.

(b) Find a recurrence relation for m_1, m_2, m_3, ...

2. (5 pts.) (a) How many onto functions are there from a set with three elements to a set with two elements?

(b) How many onto functions are there from a set with three elements to a set with five elements?

Optional Bonus Problem. (2 pts.) Who was king of England in 1202?

Optional Bonus Problem. (2 pts.) Give the name of someone else (besides Fibonacci, and the king of England) who was alive in 1202.