

**Math 301****Exam 2**

100 points possible.

1. (15 pts.) Prove the following proposition by contraposition.

**Proposition.** *Given any integer  $n$ , if  $n^2$  is even, then  $n$  is even.*

2. (10 pts.) Use the Euclidean algorithm to find the greatest common divisor of 6160 and 11000, by “hand,” showing the divisions you perform.

3. (5 pts.) Let  $a_0 = -2$ ,  $a_1 = 3$ ,  $a_2 = -5$ ,  $a_3 = 7$ ,  $a_4 = 11$ ,  $a_5 = -13$ , and  $a_6 = 17$ .

Compute  $\prod_{j=1}^3 a_{2j}$

4. (5 pts.) Simplify  $\frac{n!}{(n-3)!}$ .

5. (15 pts.) Use mathematical induction to prove that  $6^{2n} - 1$  is divisible by 7, for all integers  $n \geq 1$ .

6. (6 pts.) To say that an element is in  $(A \cap B) \cup C$  means that it is in           (a)           or in           (b)          .

(a)

(b)

7. (6 pts.) Let  $A = \{b, c, d, f, g\}$  and  $B = \{a, b, c\}$ . Find each of the following.

(a)  $A \cup B$

(b)  $B - A$

8. (6 pts.) (a) Let  $C = \{x, y, z, w\}$  and  $D = \{a, b\}$ . How many elements are in  $C \times D$ ?

(b) How many elements are in  $\mathcal{P}(\mathcal{P}(\emptyset))$ ?

9. (9 pts.) (a) Is  $\{\{w, x, v\}, \{u, y, q\}, \{p, z\}\}$  a partition of  $\{p, q, u, v, w, x, y, z\}$ ?

(b) Is  $\emptyset \in \{\emptyset\}$ ?

(c) Is  $\emptyset \subseteq \{\emptyset\}$ ?

10. (3 pts.) What is the 37th element in the one-dimensional array  $A[42]$ ,  $A[43]$ ,  $\dots$ ,  $A[100]$ ?

11. (10 pts.) A coin is tossed four times. Assume that heads and tails are equally likely on each toss. What is the probability that exactly two heads occur?

12. (10 pts.) A “control sequence” in a certain computer language consists of a backslash followed either by a single digit or by any string of one or more lower case letters. The length of a control sequence is the total number of characters in the control sequence, including the backslash.

Examples: `\5` is a control sequence of length 2. `\abbb` is a control sequence of length 5. `\53`, `\5a`, `\a6`, and `\` are four examples of strings that are *not* control sequences.

How many control sequences are there of length 4 or less?