MTH 301
Exam 2
Fall 2017

100 points possible.

1. (12 pts.) Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$. Find each of the following.

(a) $A \cup B$

(b) $A \cap B$

(c) $B - A$

(d) $A - B$

2. (12 pts.) Find the final values of j, s, and t after the following algorithm is executed. A supplemental page has the corresponding flowchart.

$j := -1$
$s := 7$
$t := 20$
while $j \neq 3$
  if $(j < 0 \text{ or } j = 2)$
    then $t := t + j$
    else $s := s + 10$
  $j := j + 1$
end while

3. (12 pts.) Let $a_0 = -2$, $a_1 = 2$, $a_2 = -1$, $a_3 = 1$, $a_4 = 2$, $a_5 = -3$, and $a_6 = 2$. Compute each of the following.

(a) $\displaystyle\sum_{i=0}^{6} a_i$

(b) $\displaystyle\sum_{j=1}^{3} a_{2j}$

(c) $\displaystyle\prod_{k=0}^{6} a_k$

(d) $\displaystyle\prod_{k=2}^{2} a_k$

4. (12 pts.) Let S be the set of all strings of a's, b's, and c's.

Define $F : S \to \mathbb{Z}$ as follows: For each string s in S

$$F(s) = \begin{cases} \text{the number of } c\text{'s to the right of the right-most } b \text{ in } s \\ 0 \quad \text{if } s \text{ contains no } b\text{'s.} \end{cases}$$

(a) Find $F(aabacac)$.

(b) Find $F(acca)$.

(c) Find $F(cba)$.

(d) What is the range of F?

5. (12 pts.) Let D be the relation defined on $\mathbb{R}$ as follows:

For all $x, y \in \mathbb{R}$, $x\, D\, y \;\Leftrightarrow\; xy \leq 0$.

(a) Determine whether or not D is reflexive. Justify your answer.

(b) Determine whether or not D is symmetric. Justify your answer.

(c) Determine whether or not D is transitive. Justify your answer.

6. (16 pts.) Here is a proof of a theorem. Fill in the blanks. (Note: Some blanks might be filled by a single variable, others by a short phrase, and others by one or more sentences.)

Theorem. *For any sets A, B, and C, $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.*

Proof. Suppose A, B, and C are any sets.
Let $x \in A \cap (B \cup C)$. *[We must show that $x \in$ ___(a)___.]* By definition of intersection, $x \in$ ___(b)___ and $x \in$ ___(c)___. Thus, $x \in A$ and by definition of union, $x \in B$ or ___(d)___.
Case 1 ($x \in A$ and $x \in B$): In this case, by definition of intersection $x \in$ ___(e)___, and so by definition of union, $x \in (A \cap B) \cup (A \cap C)$.
Case 2 ($x \in A$ and $x \in C$): In this case, ___(f)___.
Hence in either case, $x \in (A \cap B) \cup (A \cap C)$ *[as was to be shown].*
[So $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ by definition of subset.] □

(a)

(b)

(c)

(d)

(e)

(f)

7. (12 pts.) Let $3\mathbb{Z}$ be the set of all multiples of 3. That is,

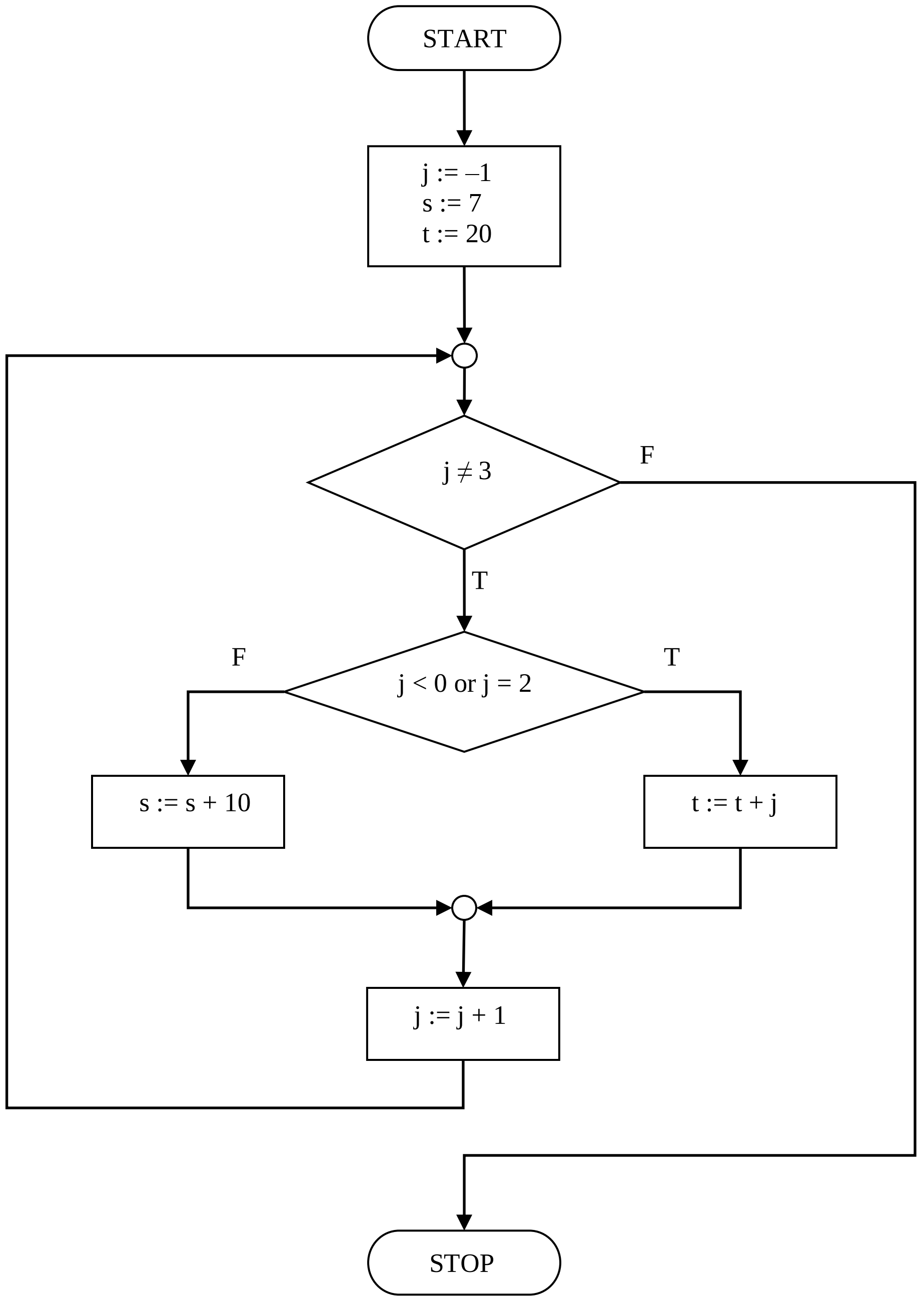
$$3\mathbb{Z} = \{n \in \mathbb{Z} \mid n = 3k \text{ for some integer } k\}.$$

Define $g : \mathbb{Z} \to 3\mathbb{Z}$ by the rule $g(n) = 3n$, for all integers n.

(a) Prove that g is one-to-one.

(b) Prove that g is onto.

8. (12 pts.) Let $c_0 = 2$, $c_1 = 10$, and for integers $k \geq 2$ let $c_k = 10c_{k-1} - 21c_{k-2}$. Prove, using strong induction, that $c_n = 3^n + 7^n$ for all integers $n \geq 0$.


START
j := –1
s := 7
t := 20
j ≠ 3
F
T
j < 0 or j = 2
F
T
s := s + 10
t := t + j
j := j + 1
STOP