MTH 301
Fall 2010

Quiz 1

1. (5 pts.) Write the truth table for this statement form.

$$(\sim p \vee r) \rightarrow (p \wedge \sim q)$$

2. (5 pts.) The attached page shows a circuit. Write an input/output table for the circuit.

3. (2 pts.) Write the negation of the following statement.

If that tree is a pine tree, then it needs to be watered.

4. (2 pts.) Write the contrapositive of the following statement.

If that iPod Touch is broken, then it can't be recharged.

5. (6 pts.) Each argument exhibits modus ponens, modus tollens, the converse error, or the inverse error. Decide whether each argument is valid or invalid, and state whether the form is modus ponens, modus tollens, the converse error, or the inverse error.

(a) If this band wears new uniforms, then it will win the championship.

This band did not win the championship.

$\therefore$ This band did not wear new uniforms.

(b) If this volleyball shirt is extra large, then it is yellow.

This volleyball shirt is yellow.

$\therefore$ This volleyball shirt is extra large.

(c) If this flower is wilting, then it is a carnation.

This flower is not wilting.

$\therefore$ This flower is not a carnation.

Quiz 2

1. (10 pts.) Prove *directly from definitions* that if n is an odd integer, then $7n+8$ is odd.

2. (10 pts.) Prove *directly from definitions* that if r and s are rational numbers, then $3r^2+5s+6$ is rational.

Quiz 3

1. (5 pts.) Find the final values of j, s, and t after the following algorithm is executed. A supplemental page has the corresponding flowchart.

```
j := 4
s := 10
t := 20
while j ≠ 8
    if (j > 6 or j = 4)
        then t := t + 1
        else s := s + j
    j := j + 1
end while
```

2. (5 pts.) Use the formula

$$M = C^d \mod pq$$

to obtain the plaintext M from the ciphertext $C = 18$, where $d = 27$, $p = 5$, and $q = 11$.

3. (5 pts.) Prove by contraposition.

For all integers n, if $6 \nmid (30+n)$, then $6 \nmid n$.

4. (5 pts.) Transform the product by making the change of variable $i = k-2$.

$$\prod_{k=6}^{n+8} \frac{k-5}{k^2+10}$$

Quiz 5

1. (5 pts.) Suppose that $b_0, b_1, b_2, \ldots$ is a sequence defined as follows:

$$b_0 = 9,\ b_1 = 27,$$
$$b_k = 7b_{k-1} - 10b_{k-2} \quad \text{for all integers } k \geq 2.$$

Prove, using strong induction, that $b_n = 3 \cdot 5^n + 6 \cdot 2^n$ for all integers $n \geq 0$.

2. (5 pts.) Here is a proof that for all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$. Fill in the blanks.

Proof. Suppose A and B are any sets and $A \subseteq B$. [*We must show that* ___(a)___.] Let $x \in$ ___(b)___. [*We must show that* ___(c)___.] By definition of union, $x \in$ ___(d)___ ___(e)___ $x \in$ ___(f)___. In case $x \in$ ___(g)___, then since $A \subseteq B$, $x \in$ ___(h)___. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in$ ___(i)___ [*as was to be shown*].

3. (5 pts.) Let S be the set of all strings of 0's and 1's, and define $h : S \to \mathbf{Z}^{nonneg}$ by

$$h(s) = \text{ the number of 1's in } s, \qquad \text{for all } s \in S.$$

(a) Is f one-to-one? Support briefly (formal proof not required), or give a counterexample.

(b) Is f onto? Support briefly (formal proof not required), or give a counterexample.

4. (5 pts.) Let $X = \{a, b, c\}$ and $\mathcal{P}(X)$ be the power set of X. A binary relation $\mathcal{R}$ is defined on $\mathcal{P}(X)$ as follows: For all $A, B \in \mathcal{P}(X)$, $A\ \mathcal{R}\ B \Leftrightarrow$ the number of elements in A is **not** equal to the number of elements in B. Determine whether the relation $\mathcal{R}$ is reflexive, symmetric, transitive, or none of these. Formal proofs are not required. You could draw the directed graph to support your answers.

Real Quiz 5

1. (5 pts.) Let $\mathcal{S}$ be the set of all strings of length 4 with characters taken from the ordinary alphabet (letters A through Z).

(a) How many strings are in $\mathcal{S}$?

(b) How many strings in $\mathcal{S}$ have no repeated characters?

2. (5 pts.) Let $B = \{1, 2, 3, \ldots, 500\}$.

(a) How many even integers are in the set B?

(b) How many odd integers are in the set B?

(c) How many subsets does B have of size 2, where the sum of the 2 integers is even?

(d) How many subsets does B have of size 2, where the sum of the 2 integers is odd?

3. (5 pts.) Suppose that six computer boards in a production run of thirty are defective. A sample of four different boards is to be selected to be checked for defects.

(a) How many different samples can be chosen?

(b) How many samples will contain at least one defective board?

(c) What is the probability that a randomly chosen sample of four contains at least one defective board?

4. (5 pts.) Use the formula

$$\binom{n}{n-2} = \frac{n(n-1)}{2} \qquad \text{(where } n \geq 2\text{)}$$

to derive formulas for the following expressions.

(a) $\binom{k+2}{k}$

(b) $\binom{n+1}{n-1}$