MTH 301
Fall 2010

Exam 1

1. (10 pts.) Write the truth table for this statement form.

$$(p \wedge r) \rightarrow (\sim p \vee \sim q)$$

2. (10 pts.) Write the negation for each of the following statements.

(a) Bridgette is a coward, and Johnie is a liar.

(b) If the cook helps to make the gluttony, then you help to make the diseases.

3. (10 pts.) Write the contrapositive for each of the following statements.

(a) If this bus is on time, then we will park in the nearby lot.

(b) If the ticket is not paid, then the car will be towed.

4. (10 pts.) Use a truth table to determine whether the argument form is valid or invalid. Clearly label the "critical rows."

$p \rightarrow r$
$q \rightarrow r$
$\therefore (p \vee q) \rightarrow r$

5. (10 pts.) Each argument exhibits modus ponens, modus tollens, the converse error, or the inverse error. Decide whether each argument is valid or invalid, and state whether the form is modus ponens, modus tollens, the converse error, or the inverse error.

(a) If Judd spends word for word with me, then I shall make his wit bankrupt.
I did not make Judd's wit bankrupt.
$\therefore$ Judd did not spend word for word with me.

(b) If Tamra's knowledge is more, then it is much darkened in her malice.
Tamra's knowledge is much darkened in her malice.
$\therefore$ Tamra's knowledge is more.

(c) If Ira is as big as a round little worm, then he is the veriest varlet that ever chewed with a tooth.
Ira is the veriest varlet that ever chewed with a tooth.
$\therefore$ Ira is as big as a round little worm.

6. (10 pts.) For the given truth table, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table. (The inputs are P, Q, and R; the output is S. The symbolic representations of the NOT-, AND-, and OR-gates are shown on the board.)

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

7. (10 pts.) Write the negation for each of the following statements.

(Move the negation "all the way inward," or "all the way right," just as we always did in class.)

(a) For all $x \in \mathbf{Z}$, if $x^2 > 9$ and $x < 0$, then $x \in A$.

(b) $\forall t \in \mathbf{R}^+$, $\exists s \in \mathbf{R}^+$ such that $\forall r \in \mathbf{Z}^+$, if $ts = r$, then either $t \neq 1$ or $s \neq 5$.

8. (10 pts.) For the Tarski World shown on a supplemental page, determine whenter each statement is true or false. LeftOf(x, y) means x is closer to the left side than y.

(a) $\exists x(\text{Triangle}(x) \wedge \forall y(\text{Square}(y) \rightarrow \text{LeftOf}(x, y)))$

(b) $\forall x(\text{Pentagon}(x) \rightarrow \exists y(\text{Square}(y) \wedge \text{LeftOf}(x, y)))$

(c) $\forall x(\text{Square}(x) \rightarrow \exists y(\text{Pentagon}(y) \wedge \text{LeftOf}(x, y)))$

9. (10 pts.) Indicate whether the arguments are valid or invalid. You may draw a diagram to support your answers, if you wish.

(a) No adapted transmitter is adventurous.

No adventurous transmitter is noisy.

$\therefore$ No adapted transmitter is noisy.

(b) No carpenters interrupt relatives.

All lumberjacks are carpenters.

$\therefore$ No lumberjacks interrupt relatives.

(c) No deplorable dentist is conniving.

My dentist is conniving.

$\therefore$ My dentist is not deplorable.

10. (10 pts.) Lewis Carroll Puzzle. Use the following letters to translate each given premise into symbols. Then deduce a logical conclusion using all given premises. Write your conclusion in symbols and in words.

(1) No kitten, that loves fish, is unteachable;
(2) No kitten without a tail will play with a gorilla;
(3) Kittens with whiskers always love fish;
(4) No teachable kitten has green eyes;
(5) No kittens have tails unless they have whiskers.

Univ. "kittens"; a = green-eyed; b = loving fish; c = tailed; d = teachable; e = whiskered; h = willing to play with a gorilla.

Exam 2

1. (5 pts.) Let c and b be integers. Define what c **divides** b means. That is, define what $c \mid b$ means.

2. (10 pts.) Find the final values of j, s, and t after the following algorithm is executed. A supplemental page has the corresponding flowchart.

$j := 3$
$s := 20$
$t := 30$
while $j \neq 7$
    if ($j > 5$ or $j = 3$)
        then $s := s + 5$
        else $t := 3t - j$
    $j := j + 1$
end while

3. (5 pts.) (a) Find 91 *mod* 8.

(b) Find 89565 *mod* 486.

4. (10 pts.) Use the formula

$$M = C^d \mod pq$$

to obtain the plaintext M from the ciphertext $C = 14$, where $d = 27$, $p = 5$, and $q = 11$.

5. (10 pts.) Find an inverse for 25 modulo 151.

6. (15 pts.) Prove by contraposition.

For all integers n, if $8n + 5$ is not odd, then n is not odd.

7. (15 pts.) Prove.

The negation of any odd integer is odd.

8. (15 pts.) Prove by mathematical induction.

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}, \quad \text{for all integers } n \geq 1.$$

9. (15 pts.) Prove that for any integer n, n^2 has the form $3k$ or $3k + 1$ for some integer k.