

Math 301
Quiz 5
Fall 2007

20 points possible. Do four of the five problems (show me which four to grade).

1. (5 pts.) Use the definition of one-to-one to prove that the following function is one-to-one.

$$f : \mathbb{Z} \rightarrow \mathbb{Z} \text{ defined by } f(x) = 5x$$

2. (5 pts.) True or false.

(a) n^2 is $O(n^5)$

(b) n^2 is $O(2^n)$

(c) n^2 is $O(n^2)$

(d) If $x, y \in \mathbb{Q}$, then $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$.

(e) If $x, y \in \mathbb{Q}$, then $\lfloor xy \rfloor = \lfloor x \rfloor \cdot \lfloor y \rfloor$.

3. (5 pts.) Use Euclid's Algorithm (i.e., the Division Algorithm) to find the greatest common divisor of 1560 and 72. Show your work!

4. (5 pts.) Prove that the square of any integer has the form $3k$ or $3k + 1$.

5. (5 pts.) Suppose $x, y, z \in \mathbb{Z}$ with $x \mid z$ and $y \mid z$. Prove that if x and y are relatively prime, then $(xy) \mid z$.