Math 301
Quiz 4
Fall 2007

20 points possible. Do three of the four problems (show me which three to grade).

1. (7 pts.) Use regular mathematical induction to prove for all positive integers n,

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=2-\frac{1}{2^n}.$$

2. (7 pts.) Given: $a_0=0$, $a_1=3$, and for $n>1$, $a_n=6a_{n-1}-9a_{n-2}$.

Use strong mathematical induction to prove $a_n=n3^n$ for all integers $n\geq 0$.

3. (7 pts.) Prove by contradiction that consecutive integers can not both be even.

4. (7 pts.) Let $\mathcal{K}$ be a set of undefined elements, and let B be an undefined relation on $\mathcal{K}$. Assume the following axioms.

Axiom 0. $\forall x,y\in\mathcal{K}$, $x\ B\ y$ or $x\ \not{B}\ y$ but not both.

Axiom 1. $\forall x\in\mathcal{K}$, $x\ B\ x$.

Axiom 2. $\forall x,y\in\mathcal{K}$, if $x\ B\ y$ and $y\ B\ x$, then $x=y$.

Axiom 3. $\forall x,y,z\in\mathcal{K}$, if $x\ B\ y$ and $y\ B\ z$, then $x\ B\ z$.

Prove:

Theorem 1. $\forall x,y,z,w\in\mathcal{K}$, if $x\ B\ y$ and $y\ B\ z$ and $z\ B\ w$ and $y\neq z$, then $x\neq w$.