

Math 301
Exam 1
Fall 2007

100 points possible.

1. (10 pts.) Definitions.

(a) Let $w \in \mathbb{Z}$. Define what it means for w to be *odd*.

(b) Let $t, p \in \mathbb{Z}$. Define what it means for t to be *divisible* by p .

(c) Let R be a relation on a set A . Define what it means for R to be *transitive*.

2. (10 pts.) Suppose $x, y, z \in \mathbb{Z}$. Prove that if $y \mid (5x + z)$ and $y \mid x$, then $y \mid z$.

3. (10 pts.) Disprove: $\forall x, y, z \in \mathbb{Z}$, if $(x + y) \mid z$ and $x \mid z$, then $y \mid z$.

4. (10 pts.) Construct the complete truth table for the given statement form.

$$((\neg x) \wedge ((\neg y) \rightarrow x)) \rightarrow y$$

5. (10 pts.) You own four different bells. You wear all four bells on your toes, but no two of the bells are on the same toe, nor are any of them on your biggest toes (i.e., the ones that went to market). In how many ways can you wear your bells? (Assume any bell will fit on any toe.)

6. (10 pts.) Complete each of the following by writing either \in or \subseteq in place of the \circ .

(a) $6 \circ \{6, 3, 0\}$

(b) $\{6\} \circ \{6, 3, 0\}$

(c) $\{6\} \circ \{\{6\}, \{3\}, \{0\}\}$

(d) $\{6\} \circ \mathbb{Z}$

(e) $2^{\{6,3,0\}} \circ 2^{\mathbb{Z}}$

7. (10 pts.) Let $m \in \mathbb{Z}$ with $5 \mid m$. Let

$$A = \{x \in \mathbb{Z} : m \mid x\} \quad \text{and} \quad B = \{x \in \mathbb{Z} : 5 \mid x\}.$$

Prove that $A \subseteq B$.

8. (10 pts.) For each statement, write the negation, but place the \neg symbol as far to the right as possible.

(a) $\forall m \in \mathbb{Z}, \forall j \in \mathbb{Z}, m + j = 1$

(b) $\forall b \in \mathbb{Z}$, if $b + 5 < 0$, then $b^2 > 25$

(c) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \exists z \in \mathbb{Z}, x + y \leq z$

9. (10 pts.) Here is a proof that for any sets A , B , and C ,

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C).$$

Fill in the blanks. (Note: Blanks (a) through (i) are very short responses, but blank (j) is longer.)

Proof. Suppose A , B , and C are particular but arbitrarily chosen sets.

Let $x \in A \cap (B \cup C)$. [We must show that $x \in$ (a) .] By definition of (b) , $x \in$ (c) and $x \in$ (d) . Thus, $x \in A$ and by definition of (e) , $x \in B$ or (f) .

Case 1 ($x \in B$): In this case, since we know $x \in A$, then by definition of (g) $x \in$ (h) , and so by definition of (i) , $x \in (A \cap B) \cup (A \cap C)$.

Case 2 ($x \in C$): In this case, since we know $x \in A$, then (j) .

Hence in either case, $x \in (A \cap B) \cup (A \cap C)$ [as was to be shown].

[So $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ by definition of subset.] ■

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j) [longer response]

10. (10 pts.) Let R be a relation on a set A . Recall that R is irreflexive if and only if $\forall x \in A, x \not R x$. Recall that R is antisymmetric if and only if $\forall x, y \in A$, if $x R y$ and $y R x$ then $x = y$. Determine if the following relation, defined on $\{1, 2, 3, 4, 5\}$, is reflexive, symmetric, transitive, irreflexive, and/or antisymmetric. (No proofs required.)

$$R = \{(1, 2), (2, 1), (3, 3), (4, 5), (5, 4)\}$$