

**Math 301****Quiz 5**

Justify all answers with neat and organized work. Clearly indicate your answers. 20 points possible.

1. (4 pts.) The formula

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

is true for all integers  $n \geq 1$ . Use this fact to solve both of these problems.

(a) If  $t$  is an integer and  $t \geq 4$ , find a formula for  $1 + 2 + 3 + \cdots + (t - 3)$ .

(b) If  $n$  is an integer and  $n \geq 1$ , find a formula for  $4 + 8 + 12 + \cdots + 4n + 8$ .

2. (3 pts.) For any nonnegative number  $c$ , define  $p_c$ , the power function with exponent  $c$ , as follows:

$$p_c(x) = x^c \quad \text{for each nonnegative real number } x.$$

Draw the graphs of the power functions  $p_4$  and  $p_5$  on the same set of axes. When  $0 < x < 1$ , which is greater:  $x^4$  or  $x^5$ ? When  $x > 1$ , which is greater:  $x^4$  or  $x^5$ ?

3. (5 pts.) Suppose a sequence satisfies the given recurrence relation and initial conditions. Find an explicit formula for the sequence.

$$a_n = 5a_{n-1} - 6a_{n-2}, \text{ for all integers } n \geq 2$$

$$a_0 = 1, \quad a_1 = 0$$

4. (4 pts.) Prove the following statement directly from the definition of  $O$ -notation. (Do not use the theorem on polynomial orders.)

$$9x^3 - 11x^2 + 3x \text{ is } O(x^3)$$

5. (4 pts.) Show that the function  $g : \mathbb{R} \rightarrow \mathbb{R}$  defined by the rule  $g(x) = -(x/4) - 8$  is decreasing on the set of all real numbers.