Math 301
Quiz 3

This take-home quiz is due Monday, October 27, at 3:00 p.m. You may turn it in earlier. Stiff penalty for late submissions. You may work together with others—open book, open notes, open talk.

Justify all answers with neat and organized work. Clearly indicate your answers. 20 points possible.

1. (5 pts.) Prove the following theorem by contraposition.

Theorem. *For all real numbers x and y, if $m = \frac{1}{2}(x + y)$, then $x \geq m$ or $y \geq m$.*

2. (5 pts.) Prove that $\sqrt{7}$ is irrational.

(You may use Exercise 11 from Section 3.6: For all integers n and all primes p, if n^2 is divisible by p, then n is divisible by p.)

3. (5 pts.) Use mathematical induction to prove that

$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}$$

for all integers $n \geq 1$.

4. (5 pts.) Determine whether the given statement is true or false. If it is true, use an element argument to prove it directly from the definitions of the set operations. If it is false, give a counterexample.

Please do your scratch work on scratch paper. For your official solution, clearly say "true" or "false," and either prove it or give a counterexample, but not both!

For all sets A, B, and C, $(A - B) \cup C = (C - B) \cup A$.